MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 92 No. 8

MEETING OF 1932 JUNE 10

Dr. H. Knox-Shaw, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

Gustavus Wynne Cook, Wynnewood, Montgomery County, Pa., U.S.A. ;
Victor Dumert, A.M.Inst.B.E., 22 Fortis Green Road, East Finchley, London, N.2 ; and
Leslie Woolven, Langcliffe, 37 Beckway Road, Norbury, Surrey.

The election by the Council of the following Associates was duly confirmed :—

Professor Giorgio Abetti, Director of the Royal Astrophysical Observatory, Florence, Italy ;
Professor Robert Emden, Professor of Physics, Technical High School, Munich, Germany ;
M. Ernest Esclangon, Director of the Paris and Meudon Observatories, France ;
Professor H. Ludendorff, Director of the Astrophysical Observatory, Potsdam, Germany ; and
Professor P. J. van Rhijn, Director of the Kapteyn Astronomical Laboratory, Groningen, Holland.

Eighty-two presents were announced as having been received since the last meeting, including—

R. G. Aitken, New General Catalogue of Double Stars within 120° of the North Pole ;

Catania Observatory, Astrographic Catalogue, vol. 6, parts 7 and 8; vol. 7, parts 7 and 8;
E. A. Milne, "The White Dwarf Stars" (Halley Lecture, 1932);
Washington, Smithsonian Institution, Annals, vol. 5.

THE SPECTRUM OF NOVA PICTORIS (1925), 1931–32.

H. Spencer Jones, F.R.S., H.M. Astronomer.

The slow decline in brightness of Nova Pictoris, 1925, has enabled the spectrum to be obtained with moderate equipment for a longer period than is customary in the case of bright novæ. A brief account of the spectrum during the period 1926–31 was published in the *M.N.*, **91**, 777–794, 1931.* The most striking feature of the spectrum in 1931 was the presence of two very strong emission bands at wave-lengths λλ 5723, 6087, which have not previously been observed. The study of the 1931 spectrum was based on an objective prism plate obtained at the Union Observatory, Johannesburg, with a total exposure of 6½ hours on 1931 February 21, 22, 23.

During the past year the decrease in brightness of the nova has been slow, and, in view of the interest of ascertaining whether these strong radiations still persisted, I asked Mr. Wood, the Union Astronomer, whether a photograph of the spectrum could be obtained this year. Mr. Wood kindly agreed, and a photograph was obtained with a total exposure of 12 hours on April 4, 5, 6, 7 by Mr. W. M. Worssell, of the Union Observatory. I am very grateful to Mr. Worssell for his co-operation. A part of the effective exposure was lost owing to a slight relative shift between the photographic and the guiding telescope, and in addition there was slight leakage of light through the back of the plate-holder, although it was carefully wrapped up. The plate was consequently not developed to the full extent and the spectrum is somewhat weak for measurement; on the other hand, the guiding was good, and in some respects the spectrum was a better one for measurement than that of the preceding year.

The spectrum this year shows a very close correspondence with that of 1931, and several faint or suspected lines which were measured on the 1931 spectrum have been confirmed by the 1932 spectrum. The two strong radiations at λλ 5723, 6087 are present with the same relative intensities as in 1931. The question whether the intensities of the two lines relative to *Hα* remain the same as in 1931 cannot be definitely settled by the 1932 plate. The plate used was a Wratten panchromatic plate, the sensitivity of which begins to fall off rapidly at λ 6100 and becomes almost zero at *Hα*; the strong *Hα* line appears on the plate as an extremely weak trace. The sensitivity of the 1931 plate did not begin to fall off until a wave-length beyond *Hα*.

* A more complete discussion of the spectrum from the date of discovery to 1931 has appeared in *Cape Annals*, **10**, pt. 9, 1931.

The wave-lengths of lines which are present or suspected on the 1931 and 1932 plates are given in the following table, with approximate relative intensities. The intensities were estimated on the 1931 plate, which is the stronger; the intensities of the additional lines, measured on the later and weaker plate, have been derived by comparison with other lines of about the same intensity, which were shown on the 1931 plate.

The Spectrum of Nova Pictoris, 1931–32

λ (1931)	λ (1932)	Mean λ	Intensity	Tabular λ	Identification
3758·9	...	3758·9	8	3759·9	O^{++}
3794·4	...	3794·4	3	3791·3	O^{++}
...	3969·6	3969·6	2	3967·5 70·1	Neb. $H\epsilon$
4101·7	4102·3	4102·0	5	4101·7	$H\delta$
4121·2	...	4121·2	2	? 4119·2	O^{+}
4288·2	...	4288·2	2		
...	4315·4	4315·4	2	? 4317·2 19·7	O^{+}
4340·7	4339·9	4340·3	12	4340·5	$H\gamma$
4363·2	...	4363·2	1	4363·7	O^{++}
4620·3	4620·1	4620·2	2		
4687·0	4687·2	4687·1	40	4686·6	He^{+}
4714·3	...	4714·3	2		
4861·2	4862·7	4861·7	15	4861·3	$H\beta$
4899·0	...	4899·0	2		
4942·7	4944·9	4943·8	8		
4968·1	...	4968·1	4		
4988·8	4989·3	4989·0	3		
5152·1	5148·5	5148·5	6		
	5163·2	5163·2	4		
5175·1	5177·5	5176·3	5		
5276·5	5278·6	5277·5	8		
5310·8	5307·9	5309·3	6		
5413·3	5414·8	5414·0	2	5411·6	He^{+}
...	5553·0	5553·0	1		
...	5683·0	5683·0	1		
5722·6	5722·6	5722·6	30		
6088·4	6086·8	6087·6	75		
6562·7	6562·1	6562·4	40	6562·8	Ha

The weak lines at λλ 3969, 4315, 5683 measured on the 1932 plate but not on the 1931 plate can just be detected on the latter, but are too diffuse for measurement. The weak line at λ 5553 is absent from the 1931 plate; the 1932 plate was independently measured four times, direct and reversed, on Hilger and Toepfer measuring machines, and this line was measured three times. It is possible that it may be spurious and due to an accidental effect of plate grain.

Identifications of most of the above lines cannot be made with certainty. It may be noted that many of them correspond approximately in wave-length with lines contained in the list of wave-lengths of bands in Wolf-Rayet stars, based on measurements of Campbell, Wright, Plaskett and Beals, given by

Beals.* This list includes the following wave-lengths : 3760, 3794, 4121, 4288, 4316, 4621, 4945, 5147, 5273, 5305, 5414, 5685. In view of the weakness of most of the detail on the two Nova Pictoris plates, and the difficulty of measurement, it seems probable that most of the fainter emissions are identical in origin with emissions in Wolf-Rayet stars, which are probably due to highly ionised elements, including oxygen and nitrogen. The two very strong radiations do not appear, however, to occur in any known Wolf-Rayet star spectrum; the lists of wave-lengths of lines, given by Campbell, Wright, Plaskett and Beals, extend in each case up to or beyond *Hα*, so that these radiations, if present, should have been detected.

The line measured at λ 5152 in 1931 was noted as forming a broad ill-defined blend with λ 5175. The weaker but sharper plate of this year seems to indicate definitely the presence of three lines ; the wave-lengths given are necessarily uncertain.

The only line included in the measures of the 1931 spectrum which, judging from its intensity, might have been expected to appear in the measures of the 1932 spectrum is the *O*III line at λ 3759. The spectrum at this wave-length is somewhat out of focus, and the line is very diffuse on the 1931 plate. A careful examination of the 1932 plate shows a very faint diffuse blur, quite impossible to measure, in the position of the λ 3759 line. This line has the high excitation potential of 36·2 volts, and its disappearance would therefore not be expected.

The decrement of the Balmer series remains high, and, so far as can be judged without *Hα* as a guide, is about the same in 1932 as in 1931. The continuous spectrum is fairly strong ; it extends in the ultra-violet to the limit of the plate, but falls off in intensity rather rapidly on the red side of λ 5700. The strongest portion on the plate is between *Hγ* and *Hδ*.

Summary.—The spectrum of Nova Pictoris in 1932 is identical in its principal features with the spectrum in 1931. The most noticeable features are the two very strong emission lines at λλ 5723, 6087, which are of unknown origin and probably due to highly ionised atoms. A number of weak emission lines present in the spectrum are probably identical with lines which have been measured in the spectra of various Wolf-Rayet stars. The continuous spectrum is fairly strong and the decrement of the Balmer series is high.

STELLAR PARALLAXES DETERMINED PHOTOGRAPHICALLY AT THE CAPE OBSERVATORY. (FOURTH LIST.)

(Communicated by H.M. Astronomer)

The present list of parallaxes of 100 stars has been derived from measures of photographs obtained with the 24-inch Victoria telescope at the Cape Observatory. The stars are numbered in rotation, in continuation of the

* " The Wolf-Rayet Stars," *Publ. Dom. Ap. Obs.*, 4, No. 17, 1930, *vide* Table I.

previous lists, which have been published in *M.N.*, **89**, 497, 1929; **90**, 190, 1929; and **91**, 822, 1931.

The Durchmusterung number of the star is given as in the Draper Catalogue, *i.e.* the B.D. number down to Dec. $-23°$, the Co.D. number south of this limit to $-52°$ and the C.P.D. number south of $-52°$. The magnitudes and spectral types, for stars contained in the Draper Catalogue, have been extracted from that catalogue. For the fainter stars they have been extracted from other sources or have been estimated from the plates.

Two values of the proper-motion in R.A. are given. The first value is based upon meridian observations or the comparison of photographic plates; the second value is the relative proper-motion derived from the parallax plates. The proper-motions in declination are based upon meridian and other determinations. Proper-motions in declination for all the stars in the list will be determined photographically at a later date, by comparison of photographs of each region with a time interval of about five years. The stars on the parallax observing programme at the Cape Observatory have been selected for the most part on account of large proper-motion; many of the published proper-motions of the fainter stars are considerably in error. In the last column, the first value of the probable error given for any star refers to the parallax and the second value to the proper-motion. These probable errors are based upon the probable error for a single plate of unit weight given in the eighth column.

The parallaxes are relative values. The mean magnitude of the comparison stars averages $11^m.0$ to $11^m.5$.

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit $''.001$	Proper-Motion Dec. Unit $''.001$	P.E. Unit Weight. Unit $''.001$	Parallax. Unit $''.001$	P.E. Unit $''.001$
				h m s					
316	$-27°$ 98	9·0	Go	0 17 0	+ 440	+ 60	19	− 10	± 7
		9·2		$-27°$ 15′·9	+ 339				± 8
317	$-27°$ 101	8·1	G5	0 17 20	+ 245	− 443	26	+ 37	± 9
		8·9		$-27°$ 34′·6	+ 189				± 9
318	Boss 127	5·71	Ko	0 32 13	+1386	− 16	29	+ 71	± 9
	$-25°$ 225	6·6		$-25°$ 19′·1	+1349				± 7
319	$-23°$ 315	7·22	G5	0 44 27	+ 502	+ 122	25	+ 55	± 8
		7·9		$-23°$ 45′·7	+ 467				± 8
320	$-26°$ 348	10·0	K2	0 59 34	− 39	− 343	14	+ 39	± 6
		11·1		$-26°$ 8′·0	− 41				± 6
321	$-13°$ 249	8·3	Ko	1 18 3	+ 394	0	24	+ 45	± 8
		9·3		$-13°$ 29′·3	+ 443				± 10
322	$-20°$ 643	8·2	K5	3 23 19	+ 521	+ 304	28	+100	± 10
		9·5		$-20°$ 9′·6	+ 546				± 7

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
323	−45° 1183	10	...	3 29 31	...	...	26	+ 9	± 10
		12·5		−45° 0′·1	+ 6				± 8
324	−45° 1184	10	...	3 29 35	− 318	+ 140	25	+ 80	± 10
		12·5		−45° 2′·8	− 336				± 8
325	−56° 578	9·0	F8	3 44 40	+ 299	...	28	+ 24	± 10
		9·7		−56° 20′·9	+ 359				± 9
326	−42° 1269	8·4	Ko	3 47 10	+ 220	+ 580	27	+ 15	± 10
		9·1		−42° 53′·1	+ 183				± 7
327	−26° 1453	10·4	F8	3 47 10	+ 315	+ 167	31	+ 16	± 11
		9·7		−26° 14′·3	+ 248				± 9
328	U.O.C. 48	...	...	4 6 31	+1102	+ 563	27	+ 53	± 11
		12·5		−53° 39′·1	+1010				± 7
329	−7° 781	9·7	A	4 10 47	−2143	−3442	28	+183	± 10
		9·7		−7° 48′·3	−2177				± 8
330	−4° 801	7·96	Go	4 15 16	...	− 300	32	− 12	± 11
		8·52		−3° 58′·9	− 3				± 11
331	−68° 268	7·8	Go	4 30 6	+ 224	+ 500	29	+ 23	± 9
		8·4		−68° 6′·1	+ 222				± 6
332	−40° 1499	9·1	F8	4 35 9	+ 223	+ 174	22	+ 22	± 8
		9·5		−40° 23′·2	+ 219				± 10
333	R Pictoris	Var.	Mα	4 43 29	...	...	26	+ 1	± 9
	−49° 1439	Var.		−49° 25′·6	+ 22				± 9
334	−5° 1044	6·00	Go	4 43 40	+ 303	− 242	20	+ 48	± 8
		6·56		−5° 50′·6	+ 299				± 6
335	U Leporis	Var.	A	4 52 0	...	...	23	+ 9	± 7
	−21° 1019	Var.		−21° 22′·4	+ 35				± 5
336	−18° 1051	5·93	Go	5 14 24	+ 380	+ 37	24	+ 61	± 9
		6·49		−18° 14′·2	+ 344				± 9
337	−60° 424	7·04	Go	5 26 27	− 310	0	27	+ 42	± 10
		7·7		−60° 29′·6	− 148				± 9
338	−31° 2652	7·9	Go	5 37 6	− 28	− 359	21	+ 36	± 7
		8·7		−31° 23′·5	− 48				± 7
339	Boss 1419	6·41	G5	5 40 16	− 297	− 352	29	+134	± 10
	−22° 1210	7·9		−22° 27′·3	− 318				± 10

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
340	π Mensæ	5·65	G5	5 45 8	+ 223	+1072	28	+ 40	± 10
	−80° 161	6·5		−80° 32′·7	+ 230				± 7
341	−46° 2096	10	...	5 54 10	+ 70	+ 491	20	+ 19	± 6
		10·9		−46° 56′·9	+ 63				± 5
342	−31° 2902	7·85	K5	5 56 36	− 398	+ 400	27	+ 51	± 9
		9·2		−31° 2′·7	− 477				± 6
343	−44° 2395	9·0	Go	5 59 17	− 43	+ 360	26	+ 14	± 8
		9·3		−44° 1′·0	− 85				± 5
344	−21° 1387	9·4	...	6 8 27	− 181	− 330	24	− 3	± 7
		10·5		−21° 43′·5	0				± 5
345	−59° 619	6·42	Go	6 14 56	− 46	− 330	23	+ 39	± 8
		7·7		−59° 10′·0	− 25				± 5
346	−45° 2481	7·5	Go	6 21 5	+ 324	+ 140	22	+ 12	± 8
		8·3		−45° 53′·8	+ 312				± 5
347	−42° 2503	6·76	G5	6 21 21	− 88	+ 740	21	+ 21	± 8
		7·4		−42° 48′·8	− 85				± 5
348	−17° 1515	7·74	G5	6 26 16	− 196	− 237	19	00	± 7
		8·52		−17° 50′·8	− 182				± 5
349	−26° 3087	9·3	...	6 27 14	+ 319	+ 143	25	+ 21	± 11
		10·0		−26° 26′·6	+ 276				± 6
350	−68° 536	8·5	Ko	6 32 29	− 131	− 480	21	+ 13	± 8
		9·5		−68° 37′·0	− 54				± 6
351	−32° 3202	7·6	G5	6 33 17	+ 318	− 40	26	+ 27	± 10
		8·3		−32° 8·3	+ 345				± 6
352	Boss 1701	5·27	G5	6 34 3	+ 101	+ 60	23	+ 24	± 9
	−32° 3216	6·9		−32° 15′·3	+ 99				± 5
353	−52° 1042	7·2	F5	6 55 28	0	− 310	29	+ 27	± 10
		7·8		−52° 30′·1	− 2				± 7
354	−74° 421	7·8	F5	6 56 4	− 40	+ 320	21	+ 38	± 8
		8·2		−74° 35′·9	− 51				± 5
355	Anon.	...	...	7 11 6	...	...	23	− 19	± 9
		11·5		−66° 54′·5	+ 19				± 6
356	U.O.C. 53	...	...	7 11 10	+ 94	− 694	29	+ 20	± 11
		11·5		−66° 56′·5	+ 120				± 8

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
357	−12° 1871	7·72	G5	7 11 16	− 488	− 154	28	+ 38	± 11
		8·50		−12° 52′·7	− 511				± 6
358	σ Puppis	3·27	K5	7 26 4	− 66	+ 180	25	+ 10	± 10
	−43° 3260	6·1		−43° 5′·9	− 43				± 7
359	−45° 3283	10	...	7 31 15	− 303	+ 434	22	+ 32	± 10
		10·5		−45° 4′·2	− 301				± 7
360	−26° 4750	10·0	Go	7 36 44	+ 342	− 282	24	+ 23	± 11
		8·9		−26° 7′·4	+ 332				± 8
361	Boss 2053	5·39	F8	7 41 51	− 255	+1691	22	+ 68	± 10
	−33° 4113	6·7		−33° 58′·5	− 286				± 7
362	Boss 2123*	5·66	F8	7 55 56	+ 502	+ 120	26	+ 59	± 10
	−59° 944	6·6		−60° 2′·1	+ 532				± 8
	Co.D. −59° 1773								
363	Co.D. −59° 1774*	9·9	,,	7 56 3	+ 465	+ 110	25	+ 47	± 9
		11·0		−60° 1′·9	+ 496				± 7
364	−31° 6673	10·3	A2	8 49 56	+ 43	+ 253	29	− 10	± 12
		9·6		−31° 38′·8	+ 14				± 8
365	−44° 5196	10	...	9 7 2	+ 224	− 330	24	+ 22	± 9
		11		−44° 40′·6	+ 253				± 7
366	−44° 5197	10	...	9 7 2	...	...	25	+ 24	± 9
		11		−44° 42′·0	− 15				± 7
367	−45° 5378	10	...	9 40 46	− 452	− 628	24	+101	± 9
		10		−45° 17′·8	− 455				± 7
368	−52° 2830	7·6	Ko	9 45 17	− 46	+ 330	21	+ 42	± 8
		8·6		−52° 9′·5	− 254				± 6
369	−26° 7505	6·32	F8	9 48 29	− 293	+ 96	25	+ 12	± 9
		7·3		−26° 51′·8	− 265				± 5
370	Boss 2704	6·28	Go	10 5 13	− 437	0	19	+ 43	± 6
	−35° 6194	6·8		−35° 22′·0	− 441				± 4
371	Boss 2718	6·44	Go	10 9 0	− 354	+ 44	23	+ 48	± 7
	−32° 7158	7·6		−32° 32′·3	− 377				± 5
372	−14° 3093	7·01	F8	10 16 0	− 210	+ 265	19	+ 43	± 7
		7·51		−14° 59′·1	− 208				± 5

* Nos. 362–3. Common proper-motion pair. The series of plates for the two stars are independent, and different reference stars were used. No. 363 = No. 233 (Third List).

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
373	Boss 2803	5·08	Go	10 27 29	− 404	+ 182	26	+ 29	± 8
	−53° 3909	5·8		−53° 12′·4	− 384				± 6
374	−11° 2916	7·9	Ko	10 31 4	+ 168	− 259	20	+ 31	± 7
		8·9		−11° 23′·3	+ 165				± 5
375	−19° 3122	7·08	G5	10 48 13	− 158	− 266	23	+ 41	± 8
		7·9		−20° 5′·3	− 152				± 5
376	−15° 3155	8·26	Ko	10 48 34	− 330	+ 39	24	− 11	± 7
		9·26		−15° 17′·4	− 310				± 5
377	−25° 8519	6·98	Go	11 7 6	+ 307	− 93	23	+ 21	± 7
		8·2		−25° 35′·5	+ 270				± 5
378	−35° 7113	8·2	Go	11 13 12	− 307	− 93	25	+ 9	± 8
		8·2		−36° 2′·8	− 206				± 6
379	−48° 6618	8·2	G5	11 29 11	− 300	− 130	27	+ 32	± 9
		8·1		−48° 15′·8	− 236				± 6
380	−43° 7228	7·4	K5	11 36 13	− 638	+ 256	29	+ 87	± 9
		8·8		−43° 51′·5	− 638				± 6
381	Boss 3110	5·96	Go	11 46 38	− 29	− 309	29	− 2	± 10
	−30° 9506	7·0		−30° 16′·2	+ 9				± 7
382	−40° 7182	7·6	G5	12 9 47	− 358	− 95	26	+ 32	± 9
		8·2		−40° 34′·9	− 317				± 7
383	−55° 5084	6·22	Ko	12 23 7	− 194	− 210	17	+ 17	± 6
		7·1		−55° 50′·9	− 242				± 6
384	−31° 9777	9·0	G5	12 29 25	− 174	− 245	24	+ 35	± 9
		9·4		−31° 19′·2	− 118				± 8
385	−12° 3709	8·1	Go	12 46 13	− 257	− 325	22	+ 23	± 8
		8·7		−12° 56′·3	− 275				± 6
386	−13° 3627	8·9	K2	12 52 31	− 510*	− 167	16	+ 58	± 6
		10·0		−13° 55′·4	− 339				± 5
387	−40° 7705	9·2	G5	13 2 58	− 497	0	21	+ 9	± 7
		10·1		−41° 6′·5	− 560				± 7
388	−34° 8720	7·57	F5	13 6 59	− 321	− 333	26	+ 20	± 8
		8·2		−34° 12′·6	− 226				± 5

* No. 386. Porter (*A.J.*, 851, 1925) states that the proper-motion is about 0″·3. The proper-motion given in Luyten's list of large proper-motions is too large.

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec. Unit ″·001	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
389	Boss 3435	6·90	Go	13 9 42	− 200	− 327	19	+ 33	± 6
	−10° 3635	7·46		−10° 49′·9	− 217				± 4
390	−58° 4943	7·4	G5	13 27 54	− 263	− 210	24	+ 17	± 7
		8·6		−58° 59′·7	− 273				± 5
391	−38° 8635	7·14	Go	13 28 40	+ 353	− 350	18	+ 50	± 6
		8·1		−38° 22′·9	+ 460				± 4
392	−56° 5963	7·9	A2	13 44 27	...	...	26	+ 12	± 7
		8·5		−56° 58′·5	− 22				± 5
393	−56° 5970	7·7	Go	13 45 4	− 237	− 420	24	+ 28	± 7
		8·5		−56° 55′·5	− 368				± 5
394	−56° 5973	8·2	B8	13 45 18	...	...	28	− 18	± 8
		8·5		−56° 57′·5	+ 2				± 6
395	−45° 8786	9·2	Go	13 47 50	− 458	0	27	+ 26	± 9
		9·4		−46° 2′·8	− 489				± 7
396	−33° 9467	8·4	Go	13 52 50	− 488	− 330	27	+ 47	± 9
		8·9		−33° 30′·3	− 453				± 6
397	−13° 3834	9·7	...	14 5 5	− 248	− 340	22	− 11	± 8
		10·5		−13° 27′·2	− 351				± 6
398	−63° 3218	9·0	Ko	14 12 59	− 171	− 480	24	+ 24	± 9
		10·0		−64° 0′·2	− 188				± 9
399	Boss 3671	5·92	F5	14 13 20	− 390	+ 369	18	+ 35	± 7
	−25° 10271	7·4		−25° 21′·8	− 403				± 7
400	−46° 9469	6·20	F8	14 32 37	− 233	− 238	19	+ 18	± 7
		6·9		−46° 8′·7	− 201				± 7
401	−56° 6368	7·42	G5	14 33 7	+ 438	− 390	23	+ 45	± 8
		8·0		−56° 35′·4	+ 405				± 8
402	−21° 4009	8·5	F8	14 54 9	− 592	− 515	20	+ 28	± 6
		9·0		−21° 36′·0	− 571				± 7
403	−46° 10348	10	...	15 39 25	...	...	23	+ 26	± 7
		11		−46° 52′·9	− 19				± 10
404	−46° 10351	10	...	15 39 30	− 402	− 386	18	+ 47	± 5
		11		−46° 54′·5	− 420				± 8
405	−83° 593	7·70	Ko	15 46 55	− 338	+ 1	28	+ 46	± 10
		8·7		−83° 57′·0	− 318				± 11

No.	Name and D.M. No.	Mag. Vis. Phot.	Type	R.A. Dec. (1900)	Proper-Motion R.A. Unit ″·001	Proper-Motion Dec.	P.E. Unit Weight. Unit ″·001	Parallax. Unit ″·001	P.E. Unit ″·001
				h m s					
406	−43° 11010	10	...	16 34 23	− 349	− 490	27	+ 89	± 9
		12·5		−43° 46′·9	− 398				± 8
407	Anon.	...	...	16 34 25	...	...	31	+ 8	± 10
		12·5		−43° 47′·7	− 12				± 10
408	−4° 4226	9·3	K5	17 0 1	− 941	−1125	29	+127	± 10
		10·4		−4° 55′·5	− 923				± 11
409	−13° 4807	9·5	Go	17 53 3	− 321	− 746	20	+ 33	± 7
		10·1		−13° 4′·0	− 470				± 9
410	−10° 4715	9·4	...	18 26 23	− 103	− 270	20	− 2	± 7
		11·0		−10° 28′·7	− 182				± 8
411	Wolf 1466	9·5	...	18 34 47	− 206	− 453	15	+ 66	± 5
		11·0		−10° 32′·6	− 163				± 7
412	−37° 13049	6·27	G5	19 0 7	− 237	− 370	17	+ 57	± 6
		6·9		−37° 57′·2	− 204				± 8
413	−45° 13178	9·9	F8	19 16 31	− 74	− 810	17	− 28	± 6
		10·2		−45° 15′·3	− 28				± 8
414	−42° 14394	9·3	Go	19 34 47	− 396*	0	20	− 9	± 6
		9·8		−42° 48′·6	− 8				± 9
415	−67° 3703	6·02	G5	19 55 32	+ 789	− 710	18	+ 54	± 6
		7·1		−67° 34′·9	+ 833				± 6
416	−50° 13237	9·2	Go	20 59 34	+ 163	− 300	28	+ 13	± 10
		9·5		−50° 21′·3	+ 177				± 7

SPECTROSCOPIC OBSERVATIONS OF THE SUN'S ROTATION MADE AT THE ROYAL OBSERVATORY, EDINBURGH, 1914–31.

J. Storey.

(Communicated by the Astronomer Royal for Scotland)

The problem of the determination of the equatorial velocity of rotation of the Sun by means of spectroscopic observations has been pursued at this observatory, using the present apparatus, since 1914. The measuring of the plates and the necessary reductions to the end of 1931 having just

* No. 414. The proper-motion determination of this star in *Cincinnati Pub.*, No. 18, 1917, is very weak. Comparison of Cape astrographic plates gives a proper-motion in R.A. of −0″·006.

been completed, it appears opportune that a summary of the results should be published.

The spectroscope consists of a Rowland concave grating of $21\frac{1}{2}$ feet radius, on loan from the University. It is mounted in the form in which the camera stands alongside the slit as described by Eagle,* and employed by Professor A. Fowler, F.R.S., in his laboratory at South Kensington.

The spectrum of the third order, about λ 6300, 1 mm. = 0·72 A., is employed, and a suitable filter mounted just behind the slit serves to eliminate the overlapping orders. The spectroscope, horizontally mounted, is fed by a siderostat which directs the Sun's rays through a 9-inch lens of 38-feet focus, giving an image of about 107 mm. (4·2 in.) in diameter.

The method of setting the Sun's image on the slit and of obtaining the desired position-angle on the Sun's limb is essentially the same as described in a previous paper.†

The exposures were made as before at fixed distances within the limb, these distances being 3·25 mm. for 1914, 4·25 mm. from 1915 to 1924 Sept. 2, and 3·32 mm. onwards, being the semi-diameters of the apertures through which the light passed.

Experiments were made prior to the commencement of this work with a system of reflecting prisms, with a view of photographing simultaneously the spectrum of opposite points on the Sun's limb, but for various reasons the method was discarded. The chief reasons, mentioned again below, are that it was found impossible to secure equal illumination of the two spectra and that the alternative method gave greater security against bias.

Attention has been concentrated on the equatorial velocity, and in practice the angle of the position circle was computed beforehand; the appropriate setting being made, an exposure was given at the west limb and immediately afterwards another at the east limb, or *vice versa*.

The plates used were Ilford special rapid panchromatic, the duration of an exposure was 3–4 minutes, and in general four or five pairs of exposures constitute a plate.

On occasion several plates were taken on the same day, but latterly only one, as it was thought advisable to spread the observations over as many days as possible.

Care was taken as to the weather conditions, and the days were chosen that were free from thin cloud, which though making very little difference to the intensity of the Sun's image, tends to scatter the light and hence give fictitious rotation values.

Particular attention was also paid to the definition of the limb, since the position of the point of observation within the limb is fundamental to the determination of the velocity.

No plate has been rejected from measurement.

With the exception of the years 1916 and 1917, when Professor Sampson made the exposures, all the plates have been taken by the writer, and all the reductions were made by him. He also measured all plates in two positions

* *Ap. J.*, **31**, 120, 1910.
† *M.N.*, **71**, 674. 1911.

in the micrometer except for 1915, when he measured in one position only; but on his departure in 1915 August for war service the whole of the plates for this year were measured in both positions by Misses S. A. and I. C. Falconer, who were experienced measurers of the astrographic catalogue plates. The results of the three are in very good agreement.

For the earlier years a Hilger measuring machine, type L1, was used, and for the later years, type L18. In each case one turn of the micrometer screw was 1 mm.

In all 22 lines were measured (Table I), 9 solar and 13 telluric, the position of the former being determined differentially with respect to the latter. The solar lines used are the same as in the previous paper, while the telluric have the addition of two, λλ 6299·231 (3) and 6319·600 (0).

Table I

λ	Element	Intensity	λ	Element	Intensity
6280·396	*A*o	2	6309·896	*A*o	2
80·628	*Fe*	3*d*	10·643	*A*o	1
81·185	*A*o	1	11·511	*Fe*	1
90·224	*A*o	2	14·244	*A*o	0
90·981	*Fe*	4	14·675	*Ni*	4
92·170	*A*o	2	14·983	*A*o	0
			15·822	*Fe*	1
95·966	*A*o	3	18·036	*Fe*	6
97·808	*Fe*	5	19·600	*A*o	0
99·231	*A*o	3			
6301·517	*Fe*	7			
02·005	*A*o	2			
02·508	*Fe*	5			
02·771	*A*o	2			

In the reductions the measurements were divided into six groups as shown by the bars in Table I, and the value of the displacement for a solar line was determined differentially from the mean of the adjacent telluric lines in that group.

This method was adopted principally to counteract the possibility of an undetected inclination of the vertical micrometer wire to the direction of the spectrum lines, which, if present, introduces a serious error into the measurements over a long run of the screw.

The annual means were then formed, the daily value being weighted according to the number of observations on each day. These means $(v + v_1)$ are shown in Table II, together with the daily angular velocity (ξ) and the corresponding period in days. For purposes of comparison the values

obtained at Mount Wilson * are given in column 7, and in column 8 various determinations at the other observatories indicated.

Table II

Year	Number of		$v+v_1$	ξ	Period	Mt. Wilson $v+v_1$	Various Values	
	Days	Obsns.					$v+v_1$	Place
			km./sec.	°	days	km./sec.	km./sec.	
1914·79	11	68	1·923	13·69	26·30	1·92		
15·40	10	70	1·923	13·69	26·30	1·96	1·95	Ottawa
16·62	6	31	1·976	14·07	25·59	1·91		
17·46	2	10	1·941	13·82	26·05	1·94		
...	...	...	...	...	...	1·95		
19·68	8	48	2·020	14·38	25·03	1·90		
20·52	9	89	1·992	14·18	25·39	1·90		
21·61	8	43	1·962	13·96	25·79	1·91		
22·61	10	48	1·997	14·21	25·33	1·91		
23·31	1	5	2·064	14·70	24·49	1·89	1·93	Kodaikanal
24·48	7	30	1·988	14·15	25·44	1·91		
25·51	9	43	1·954	13·91	25·88			
26·62	10	46	1·960	13·94	25·83			
27·66	6	25	1·991	14·17	25·41			
28·68	8	29	1·994	14·19	25·37	1·89	1·99	Arcetri
29·44	13	52	1·958	13·94	25·83	1·92		
30·75	7	28	1·926	13·71	26·26	1·95	1·85 1·89	Arcetri Potsdam
31·65	12	44	2·010	14·31	25·16	...	2·02	Ewhurst

The striking feature of this table is the lack of agreement between the various determinations, and in particular the almost systematic differences of the Mount Wilson figures from those obtained here ; they show nearly always a smaller velocity, while the other results in column 8 lend little support to either the one or the other.

In looking for a possible source of these differences the method of observation which has already been described naturally comes under review.

It can confidently be said that the setting of the Sun's disc in front of the slit presented little difficulty in control.

The method excludes any question of the unequal illumination of the grating between one limb and the other, which appears to be a problem when reflecting prisms are used.

Also, in measuring, the spectrum of only one limb is seen in the micrometer field at any time, so that there is no question of the measurer being biased by the appearance of the corresponding line in close proximity.

The question of the personality of the measurer has been tested by

* St. John, *Int. Research Council, Third Report, Solar and Terrestrial Relationships*, p. 121, 1931.

Plaskett and De Lury.* To test the matter here, several plates were measured by Dr. E. A. Baker in the Koch microphotometer, the settings being thus entirely automatic and capable of repetition within 0·001 mm. The mean rotation values thus found agreed within 1 per cent. with those determined by eye measurement.

Incidentally it tested the symmetry of the lines used. The only line (λ 6309·896 *Ao*) in which dissymmetry varying with the limb was evident showed that apparently there was a very faint solar line of practically the same wave-length, but it could not be detected visually.

As has already been indicated, the observing and the measuring have, with few exceptions, been the work of the same person, so that the material should be homogeneous.

That large differences in the rotation value occur from plate to plate and often on the same plate is well known,† and a recent paper by Abetti and Righini,‡ who, from observations at Mount Wilson in 1930 August–October, obtained very different values from the east and west limb observations, using a certain class of lines, shows the uncertainties of the problem.

With regard to the rotation values given by individual lines in the present series, the deviation of each line from the mean value for all the nine lines is as follows :—

λ	6280·6	6290·9	6297·8	6301·5	6302·5	6311·5	6314·6	6315·8	6318·0
Element	*Fe* 3	*Fe* 4	*Fe* 5	*Fe* 7	*Fe* 5	*Fe* 1	*Ni* 4	*Fe* 1	*Fe* 6
km./sec.	−·065	+·106	−·027	−·005	−·009	−·013	+·005	+·003	+·001
Degrees/day	−·46	+·75	−·19	−·04	−·06	−·09	+·04	+·02	+·01

All these lines apparently represent low levels in the reversing layer, but it is evident from the above that there is some cause operating by which the first two or possibly three lines individually give rotation values different from the rest.

These differences suggest that the divergence of the rotation values obtained by various observers may partly be due to this cause, though in the past the differences shown by individual lines in the reversing layer of the Sun have been generally small.§

With regard to the accuracy of the results, the probable error for a day or plate is ± ·024 km. and for a year ± 0·008 km., fairly constant from year to year.

* *Trans. R. Soc. Canada*, **6**, 29, 1912.
† Evershed, *M.N.*, **92**, 106, 1931.
‡ *Pubbl. Osserv. Arcetri*, Fas. **49**, 25, 1931.
§ Adams, *Papers Mt. Wilson Obs.*, I, 1, 1911.

MEAN AREAS AND HELIOGRAPHIC LATITUDES OF SUNSPOTS IN THE YEAR 1931.

(Communicated by the Astronomer Royal)

The results given here are in continuation of those in the *M.N.*, **91**, 1004, 1931, and are derived from the measurement of photographs taken at the Royal Observatories of Greenwich and the Cape, and in India at the Kodaikanal Observatory.

TABLE I

No. of Rotation	Rotation Commenced G.C.T.	No. of Days on which Sun was photographed	Mean of Daily Areas					
			Projected *			Corrected for Foreshortening †		
			Umbræ	Whole Spots	Faculæ	Umbræ	Whole Spots	Faculæ
1034	Jan. 1·30	28	39	208	759	35	199	881
1035	28·64	27	200	999	604	129	638	720
1036	Feb. 24·98	27	190	971	878	147	775	1080
1037	Mar. 24·30	28	111	561	983	81	410	1176
1038	Apr. 20·57	27	77	323	846	66	275	1014
1039	May 17·81	27	105	492	790	71	334	956
1040	June 14·01	27	32	138	900	27	123	1058
1041	July 11·21	28	30	113	750	20	76	874
1042	Aug. 7·42	27	25	108	447	17	76	532
1043	Sept. 3·67	27	40	173	538	33	150	608
1044	30·94	27	26	124	379	18	88	433
1045	Oct. 28·23	28	50	213	459	47	188	545
1046	Nov. 24·53	27	110	495	526	67	302	593

TABLE II

Year	No. of Days		Mean Daily Areas					
	Photographed	Without Spots	Projected.*			Corrected for Foreshortening †		
			Umbræ	Whole Spots	Faculæ	Umbræ	Whole Spots	Faculæ
1923	365	171	15	74	189	10	55	222
1924	366	97	62	361	498	46	276	575
1925	365	19	189	1102	1508	141	830	1750
1926	365	2	284	1651	2159	217	1262	2556
1927	365	0	248	1417	1812	186	1058	2212
1928	366	0	333	1884	2134	244	1390	2589
1929	365	0	334	1695	2190	245	1242	2567
1930	365	4	145	705	1378	106	516	1630
1931	365	42	79	372	679	57	275	801

* Expressed in millionths of the Sun's disc.

† Expressed in millionths of the Sun's hemisphere.

Table I gives the mean daily areas of umbræ, whole spots and faculæ for each synodic rotation of the Sun included in the year 1931, whilst the mean values for each year as a whole from 1923 to 1931 are given in Table II.

Table III gives for each rotation in the year 1931 the mean daily area of the whole spots (corrected for foreshortening) and the mean heliographic latitude of the spotted areas for both hemispheres, northern and southern. The mean heliographic latitude of the entire spotted area and the mean distance from the equator of all spots are also tabulated. The mean values for the years 1923 to 1931 are given in Table IV. Tables II and IV are in continuation of similar tables given in the *M.N.*: for the years 1874 to 1888—**49**, 381 and 382; 1889 to 1902—**63**, 465 and 466; 1901 to 1914—**76**, 402 and 403; 1913 to 1924—**85**, 1007 and 1008.

TABLE III

No. of Rotation	Rotation Commenced G.C.T.	Spots NORTH of the Equator		Spots SOUTH of the Equator		Mean Latitude of entire Spotted Area	Mean Distance from Equator of all Spots
		Mean Daily Area	Mean Heliographic Latitude	Mean Daily Area	Mean Heliographic Latitude		
			°		°	°	°
1034	Jan. 1·30	169	6·89	30	18·30	+ 3·08	8·62
1035	28·64	539	7·10	99	5·09	+ 5·21	6·79
1036	Feb. 24·98	539	5·68	237	7·91	+ 1·53	6·36
1037	Mar. 24·30	261	8·64	149	6·65	+ 3·09	7·92
1038	Apr. 20·57	245	9·35	30	7·13	+ 7·57	9·11
1039	May 17·81	210	6·46	124	15·65	− 1·73	9·86
1040	June 14·01	58	7·24	65	9·65	− 1·68	8·51
1041	July 11·21	31	6·95	45	8·39	− 2·17	7·81
1042	Aug. 7·42	50	7·04	26	9·64	+ 1·25	7·95
1043	Sept. 3·67	105	11·04	45	5·90	+ 5·98	9·50
1044	30·94	25	17·02	63	14·62	− 5·59	15·31
1045	Oct. 28·23	147	10·08	42	7·20	+ 6·26	9·44
1046	Nov. 24·53	300	10·49	2	0·81	+10·41	10·42

TABLE IV

Year	No. of Days on which Sun was photographed	Spots NORTH of the Equator		Spots SOUTH of the Equator		Mean Latitude of entire Spotted Area	Mean Distance from Equator of all Spots
		Mean Daily Area	Mean Heliographic Latitude	Mean Daily Area	Mean Heliographic Latitude		
			°		°	°	°
1923	365	33	13·88	22	17·32	+ 1·34	15·26
1924	366	232	22·19	45	25·54	+14·49	22·73
1925	365	517	20·70	313	19·37	+ 5·60	20·20
1926	365	663	19·27	599	17·98	+ 1·60	18·66
1927	365	379	17·75	679	13·54	− 2·33	15·05
1928	366	727	11·93	663	15·23	− 1·02	13·50
1929	365	656	10·73	586	10·27	+ 0·82	10·51
1930	365	286	10·60	230	8·96	+ 1·88	9·87
1931	365	200	7·91	75	9·39	+ 3·21	8·31

The rotations in Tables I and III are numbered in continuation of Carrington's series (*Observations of Solar Spots made at Redhill*, by R. C. Carrington, F.R.S.), No. 1 being the rotation commencing 1853 November 9. The assumed prime meridian is that which passed through the ascending node of the Sun's equator on the ecliptic at mean noon on 1854 January 1, and the assumed period of the Sun's sidereal rotation is 25·38 days.

The principal features of the record for the year 1931 are as follows :—

1. The continued decline of solar activity towards the minimum phase of the 11-year cycle, the mean daily areas of sunspots and faculæ being about half those for the previous year.

2. On 42 days no spots were seen on the Sun's disc. On 11 days no faculæ were measured near the limbs.

3. The northern hemisphere was the more active.

4. The mean latitude of spots (8°·3) shows a decrease of $1\frac{1}{2}°$ as compared with that for the year 1930. Latitudes of spots ranged from 1° to 26°.

5. The number of groups lasting for two days or more was 116 ; of these, 76 were in the northern hemisphere and 40 in the southern. Besides these, 51 very small groups (usually consisting of one or two tiny spots) were measured on one day only ; 34 were in the northern hemisphere and 17 in the southern.

Royal Observatory, Greenwich :
1932 *June* 2.

REDUCTION OF SPECTROSCOPIC ABSOLUTE MAGNITUDES TO A UNIFORM SYSTEM.

P. J. van Rhijn.

1. *Introduction and Summary*

The present paper deals with the reduction of spectroscopic absolute magnitudes * to a uniform system. The reduction was needed for an investigation of the distribution of the densities of the stellar system according to distance from the Sun, galactic longitude and galactic latitude. The adopted system of absolute magnitudes is derived mainly from trigonometric parallaxes and mean parallaxes of stars of given apparent magnitude and proper motion $\pi_{m,\mu}$ of *Gron. Publ.*, No. 34, 1923, slightly corrected in accordance with some recent data. The trigonometric parallaxes have been reduced to the system of *Gron. Publ.*, No. 34, by the application of the corrections of Table III of *Gron. Publ.*, No. 37, 1925. It has been shown in the latter paper that this system is practically identical with the Allegheny trigonometric parallaxes.

The errors in the trigonometric parallaxes depending on the right ascension † have been taken into account.

* Absolute magnitude in this paper means the apparent magnitude at a distance of 10 parsecs.

† F. Schlesinger, *General Catalogue of Stellar Parallaxes*, pp. iv–vi, 1924 ; A. van Maanen, *Contributions from the Mount Wilson Observatory*, No. 357, 1928.

The results derived from the mean parallaxes $\pi_{m,\mu}$ and trigonometric parallaxes have been checked by the absolute magnitudes found from the parallactic motion and from the comparison of peculiar angular and linear motions respectively.

The absolute magnitudes of the B stars cannot be found by means of the above methods : their mean parallaxes $\pi_{m,\mu}$ have not been computed in *Gron. Publ.*, No. 34, and the trigonometric values are too small to be of much use. We have reduced the absolute magnitudes of the B stars to the values given by Kapteyn in *Mount Wilson Contributions*, Nos. 82 and 147.

All systems of spectroscopic absolute magnitude with a sufficient number of stars have been considered with the exception of the Upsala values, for which, probably on account of the small dispersion of the spectra, the probable error of the absolute magnitudes is larger than for the other authorities.

The adopted corrections are found in Table IV.

It is well known that the absolute magnitudes derived by means of the spectroscopic criteria are not the true absolute magnitudes ; the spectroscopic absolute magnitudes of stars of a given true absolute magnitude are distributed according to some frequency law or other.

The difference between the observed spectroscopic absolute magnitude reduced to a uniform system and the true absolute magnitude is therefore composed of two factors :

(*a*) The accidental error of observation of the ratio of the line intensities from which the absolute magnitude is derived.

(*b*) The cosmic error of the spectroscopic absolute magnitude, *i.e.* the difference between the spectroscopic magnitude freed from the error (*a*) and the true absolute magnitude. The cosmic error is caused by the fact that other factors besides the absolute magnitude have an influence on the ratio of the line intensities.

It will be shown that the errors (*a*) and (*b*) together are distributed in accordance with a Gaussian curve, at least in the case of the dwarf stars. The same curve is assumed to hold for other classes of stars. The probable deviations of the curves are found in Table VII.

The spectroscopic criteria have been reduced in this paper to absolute magnitudes by means of the condition :

that the mean spectroscopic criterion of stars of a given true absolute magnitude corresponds with this absolute magnitude. (1)

2. *Methods Used for the Reduction*

The following methods have been used for the reduction of the spectroscopic absolute magnitudes to a uniform system :—

(a) *Trigonometric Parallaxes.*—This method is illustrated by means of the Victoria * absolute magnitudes M_V. The stars have been divided into

* R. K. Young and W. E. Harper, *Publications of the Dominion Astrophysical Observatory*, 3, No. 1, 1924.

groups of absolute magnitude by means of the Mount Wilson * values, and the mean Victoria absolute magnitude $\bar{M}_V$ as well as the mean trigonometric parallax $\bar{\pi}_t$ have been formed for each group. The mean absolute magnitude $\bar{M}_t$ of the group has been derived from the mean trigonometric parallax. The correction to the mean Victoria absolute magnitude is $\bar{M}_t - \bar{M}_V$.

The stars cannot be divided into groups by means of the Victoria absolute magnitudes because by doing so the accidental errors of the spectroscopic absolute magnitudes of a group would not cancel out in the mean. Using the Mount Wilson values as a basis of division the accidental error of the Victoria absolute magnitude will practically be zero if the errors of the Victoria and Mount Wilson values are independent.† Therefore the correction to the Victoria absolute magnitudes is simply $\bar{M}_t - \bar{M}_V$.

Let $F(M)dM$ be the distribution of the true absolute magnitudes of the stars with known Mount Wilson and Victoria absolute magnitudes;

M_V be the Victoria absolute magnitude reduced to a uniform system;

M_W be the Mount Wilson absolute magnitude reduced to a uniform system;

$f(M_V - M)dM_V$ be the frequency of the Victoria absolute magnitudes of stars of true absolute magnitude M;

$f(M_W - M)dM_W$ be the frequency of the Mount Wilson absolute magnitudes of stars of true absolute magnitude M.

The mean true absolute magnitude of stars selected on the basis of Mount Wilson absolute magnitude M_W is:

$$\bar{M} = \frac{\int_{-\infty}^{+\infty} MF(M)f(M_W - M)dM}{\int_{-\infty}^{+\infty} F(M)f(M_W - M)dM}. \qquad (2)$$

Further, the mean Victoria absolute magnitude of the same group of stars is

$$\bar{M}_V = \frac{\int_{-\infty}^{+\infty} M_V dM_V \int_{-\infty}^{+\infty} F(M)f(M_V - M)f(M_W - M)dM}{\int_{-\infty}^{+\infty} dM_V \int_{-\infty}^{+\infty} F(M)f(M_V - M)f(M_W - M)dM}. \qquad (3)$$

If we change the order of integration and remember that according to (1)

$$M = \frac{\int_{-\infty}^{+\infty} M_V f(M_V - M)dM_V}{\int_{-\infty}^{+\infty} f(M_V - M)dM_V},$$

we find that $\bar{M}$ of formula (2) is equal to $\bar{M}_V$ of formula (3).

* W. S. Adams, A. H. Joy, G. Strömberg and Cora G. Burwell, *Contributions from the Mount Wilson Observatory*, No. 199, 1921.

† It can be shown that the Victoria absolute magnitudes reduced according to this method fulfil condition (1) if the cosmic and observational errors of the Mount Wilson and Victoria absolute magnitudes are independent.

The reduction by means of the trigonometric parallaxes has always been performed according to the principles outlined in this example. The stars are grouped on the basis of a set of absolute magnitudes different from that for which the correction is required.

The trigonometric parallaxes used are the means of the values determined by the photographic method with a long-focus telescope. The Dearborn parallaxes have not been used on account of the large systematic correction.* The relative parallaxes have been reduced to absolute values by means of Table VII of *Gron. Publ.*, No. 45, 1931. Systematic mean corrections have been applied to each set of parallaxes separately in accordance with Table III of *Gron. Publ.*, No. 37. The dependence of the correction on the right ascension has not been considered in the paper quoted. We have adopted the mean of the variations of the correction with the right ascension, as found by Schlesinger † and van Maanen,‡ the mean of the corrections for all hours of right ascension being reduced to the values of Table III of *Gron. Publ.*, No. 37.

No correction has been derived in *Gron. Publ.*, No. 37, for the Greenwich parallaxes. The corrections given by van Maanen in Table X of the publication quoted, diminished by 0″·002, have been adopted.

The systematic corrections to the parallaxes of the Cape Observatory and the Yale southern station have not yet been investigated ; no correction has been applied to these sets of parallaxes.

The mean value of the trigonometric absolute magnitude for a group is

$$\bar{M}_t = \bar{m} + 5 + 5\,\overline{\log \pi_t}\,, \qquad (4)$$

where m is the apparent magnitude. It has been found by adding the correction

$$5\,(\overline{\log \pi_t} - \log \bar{\pi}_t) \qquad (5)$$

to the quantities $\bar{m} + 5 + 5 \log \bar{\pi}_t$ which are easily computed from the data of observation.

The correction (5), which varies between $-0{\cdot}2$ and $-0{\cdot}6$ for the different groups, has been derived from a provisional distribution of the true parallaxes of each group, computed by means of an adopted distribution of the densities and the absolute magnitudes.

(b) *Secular Parallaxes.*—The method of grouping is the same as in the case of the trigonometric parallaxes. The mean parallax of the group has been computed according to the formula

$$\bar{\pi} = f \frac{\Sigma v \sin \lambda}{\Sigma \sin^2 \lambda},$$

where

v is the component of the proper motion in the direction of the antapex of the solar motion;

λ is the angular distance between the star and the solar apex;

f is a constant factor depending on the Sun's velocity.

* *Gron. Publ.*, No. 37, Table III, 1925.

† *General Catalogue of Stellar Parallaxes*, pp. iv–vi, 1924.

‡ *Contributions from the Mount Wilson Observatory*, No. 357, 1928.

The proper motions used in this and the following sections are from Boss's *Preliminary General Catalogue*. The corrections in μ_δ derived by Boss and Jenkins have been applied;* the proper motions have further been corrected for the galactic rotation and precession in accordance with Oort's paper, *Bulletin of the Astronomical Institutes of the Netherlands*, **132**. The adopted values of the constants are :

$$\frac{A}{4{\cdot}74} = +0''{\cdot}0025\,; \qquad \frac{B}{4{\cdot}74} = -0''{\cdot}0035\,; \qquad l_0 = 325^\circ\,;$$

$$\Delta e + \Delta\lambda = +0''{\cdot}0117\,; \qquad \Delta p = +0''{\cdot}0113.$$

The notation is the same as in the quoted paper by Oort.

The adopted value of $\frac{A}{4{\cdot}74}$ is somewhat larger than Oort's result for the angular motions ; the reason is that the radial velocities give a higher value than the proper motions.

The adopted value for the solar motion throughout this investigation is

$\alpha = 270^\circ.$
$\delta = +31^\circ.$
$V = 19{\cdot}5$ km./sec. for the F to M stars.
$V = 16{\cdot}0$ km./sec.† for the A stars.

The method has been applied to the groups of absolutely very bright stars for which the trigonometric absolute magnitudes are less reliable. I have convinced myself by comparison with Table XII of *Gron. Publ.*, No. 30, 1920, that the stars to which the method has been applied have not been selected on the basis of either large or small proper motion.

The mean values of the absolute magnitude have been computed from the mean parallaxes as in the case of trigonometric parallaxes.

The stars have been grouped on the basis of a set of absolute magnitudes different from that for which the correction is required.

(*c*) In order to check the large negative corrections to the Mount Wilson and Victoria absolute magnitudes of absolutely bright stars I have computed the parallaxes of these stars by means of the formula

$$\bar{\pi} = 4{\cdot}74\frac{\bar{\tau}}{\bar{V}}, \tag{6}$$

where

τ is the component of the annual proper motion perpendicular to the direction of the solar motion ;
V is the radial velocity in km./sec. freed from the Sun's motion.

The method of grouping is the same as that described in the section on the trigonometric parallaxes.

It is easily shown that the angle between the τ component of proper motion and the line of the true vertices of Kapteyn's star streams is always

* *The Astronomical Journal*, **37**, 177, 1927.
† *Gron. Publ.*, No. 37, p. 29, 1925.

between 50° and 90°. The angular and linear motions in the formula (6) are therefore only comparable if the radial velocities are restricted to stars the angular distances of which from either of the true vertices lie between the same limits. Only those stars have, therefore, been used in the determination of the mean velocities $\overline{V}$.

The dependence of the mean velocities of the stars on the absolute brightness has been taken into account by a selection of stars of the same spectroscopic brightness in the case of the linear motions as for the angular motions.

The radial velocities have been corrected for differential galactic rotation.

For the grouping of the stars and the computation of $5\,(\overline{\log \pi} - \log \bar{\pi})$ see the section on trigonometric parallaxes.

(*d*) The mean parallaxes of stars of determined proper motion, apparent magnitude, galactic latitude and spectral class derived in *Gron. Publ.*, No. 34, have also been used for the reduction of the spectroscopic absolute magnitudes.

The stars have been grouped on the basis of the proper motion and the apparent magnitude. The mean parallax $\bar{\pi}_{m,\mu}$ and the corresponding value of $m + 5 + 5 \log \bar{\pi}_{m,\mu}$ have been computed for each star by means of Tables XXXIV to XXXVIII of *Gron. Publ.*, No. 34. The mean of the values of $m + 5 + 5 \log \bar{\pi}_{m,\mu}$ has been taken for all stars of a group and the mean absolute magnitude $\overline{M}$ has been found by the addition of $5\,(\overline{\log \pi_{m,\mu}} - \log \bar{\pi}_{m,\mu})$. The latter quantity has been computed in *Gron. Publ.*, No. 34, p. 60, as a function of h which is given for each spectral class in Table XLII. The value actually adopted is somewhat nearer to zero than the value of *Gron. Publ.*, No. 34. Further, a small correction has been applied which takes account of the accidental errors in the proper motions; see Table XXV of *Gron. Publ.*, No. 30.

The $\bar{\pi}_{m,\mu}$ used in this paper are the values of *Gron. Publ.*, No. 34, corrected in accordance with a revised computation necessary for the following reasons:

(α) The distribution of the proper motions and the radial velocities from which the $\bar{\pi}_{m,\mu}$ are derived is modified by the correction for galactic rotation and precession quoted above.

(β) The spectral classes used in *Gron. Publ.*, No. 34, are derived from *Harvard Annals*, **28**, **56** or **50**, with a preference for the two former catalogues. It appears that the spectral classes of the new Henry Draper Catalogue * differ from the earlier adopted classification. I do not know whether the differences have a systematic character but there is no doubt that the distribution of the proper motions of the fainter Boss stars of a given spectral class adopted in *Gron. Publ.*, No. 34, is modified by the spectral data of the Henry Draper Catalogue.

(γ) The secular parallaxes of the fainter Boss stars of a given spectral class must be corrected in accordance with the new data of the Henry Draper Catalogue.

All trigonometric parallaxes determined by photographic methods and

* *Harvard Annals*, **91–99**, 1918–1924.

available in 1931 March have been included in the new solution of the $\bar{\pi}_{m,\mu}$.

(*e*) The corrections to the spectroscopic absolute magnitudes have been computed from the following moving-cluster parallaxes :—

From the moving clusters investigated by Rasmuson * only the Taurus, the Ursa Major, the Scorpio-Centaurus and the Orion groups have been used.

The Pleiades have been excluded because the absolute magnitudes of the A stars in this cluster are probably different from the values found in other clusters and for the A stars in the neighbourhood of the Sun.† The Perseus cluster has not been used because the stream velocity needed for the computation of the parallax is very uncertain, most of the members of the cluster having an angular distance from the apex of the motion which does not differ much from 90°. The parallaxes of the Taurus and Ursa Major clusters are very valuable material for the reduction of the absolute magnitudes of the A stars. In the case of the Taurus cluster I have used the stars given in Rasmuson's paper, the P.G.C. numbers of which in column 1 are not bracketed ; in the case of the Ursa Major cluster only the P.G.C. numbers 553, 1024, 1478, 1732, 2813, 2930, 3117, 3190, 3363, 3474, 3475, 3480, 3506, 3798, 3961, 4009, 5573 have been used.‡

The parallaxes of the cluster stars given by Rasmuson have been adopted. A correction of $+1{\cdot}57$ has been applied to his absolute magnitudes in order to reduce them to the zero-point of the present paper.

The Scorpio-Centaurus cluster and the Orion group are very valuable material for the reduction of the absolute magnitudes of the B stars. The clusters were investigated by Kapteyn,§ and I have adopted his parallaxes and absolute magnitudes. The values of π_2 of *Mount Wilson Contributions*, No. 82, 1914, which have been corrected for systematic errors, have been used. The basis of grouping is again a set of absolute magnitudes different from that for which the correction is required.

(*f*) In the first and second methods of reduction the supposition has been made that the errors in two systems of spectroscopic absolute magnitudes are independent. This is true for the errors of observation in the line intensities, but it seems rather improbable for the cosmic errors in the absolute magnitudes. I have therefore derived the correction to the Victoria and the Mount Wilson absolute magnitudes, and to some sets of A stars, according to a method which is independent of this supposition. The Mount Wilson corrections thus found are practically the same as those derived according to the methods (*a*) and (*b*). The conclusion is that the interdependence of the cosmic errors of the different sets of absolute magnitudes has not seriously influenced our results.

The method used for the derivation of the correction to the spectroscopic absolute magnitudes for the Victoria results is as follows : The stars

* *Meddelanden från Lunds Astronomiska Observatorium*, série II, No. 26, 1921.

† See also Hertzsprung, *M.N.*, **89**, 675, 1929.

‡ Bottlinger, *Astronomische Nachrichten*, **198**, 153, 1914.

§ *Mount Wilson Contributions*, No. 82, 1914, and No. 147, 1918.

were divided into groups on the basis of the Victoria absolute magnitudes and the mean value was formed for each group. This mean value does not represent the true average : on account of the accidental errors in the spectroscopic absolute magnitudes, both observational and cosmic, the absolutely bright groups are too bright and the faint ones too faint.

We have computed the correction for the mean spectroscopic absolute magnitude as follows : The distribution of the true absolute magnitudes of the Victoria stars of a given spectral class is found from the density and luminosity curve of *Gron. Publ.*, No. 38. The spectroscopic absolute magnitudes of stars of given true absolute magnitude are assumed to be distributed according to a Gaussian curve with a probable deviation given in Table VII. We thus find the distribution of the Victoria stars according to true absolute magnitude and spectroscopic absolute magnitude. From this distribution table the mean value of the true absolute magnitude M_t for stars of a given spectroscopic absolute magnitude M_s is easily computed. The differences $M_t - M_s$ represent the corrections to the mean Victoria absolute magnitudes due to the accidental errors, both observational and cosmic, of the latter.

The mean absolute magnitude derived from the trigonometric and secular parallaxes, respectively, minus the mean Victoria absolute magnitude, corrected as described above, represents the systematic correction to be applied to the Victoria absolute magnitudes.

3. Systematic corrections have been computed for the following catalogues :—

B and A Stars

		Abbreviation	Observatory
Adams and Joy	*Mount Wilson Cont.*, No. 244	M.W. 244	Mount Wilson
Adams and Joy	*Mount Wilson Cont.*, No. 262	M.W. 262	,, ,,
Edwards	*Monthly Notices*, **84**, 367	N.L. 3	Norman Lockyer
Edwards	*Monthly Notices*, **85**, 440	N.L. 4	,, ,,
Edwards	*Monthly Notices*, **87**, 364	N.L. 5	,, ,,
Edwards	*Monthly Notices*, **88**, 175	N.L. 6	,, ,,
Edwards	*Monthly Notices*, **90**, 523	N.L. 7	,, ,,
Abetti	*Pubbl. Arcetri-Firenze*, No. 42	Arc. 42	Firenze
Rimmer	*Mem. Mount Stromlo*, No. 2	M.S.	Mount Stromlo
Douglas	*Astrophysical Journ.*, **64**, 262	Ye.	Yerkes
Woods	*Monthly Notices*, **87**, 387	N.L. 8	Norman Lockyer

F to M Stars

Adams, Joy, Strömberg, Burwell	*Mount Wilson Cont.*, No. 199	M.W. 199	Mount Wilson
Rimmer	*Memoirs Royal Astronomical Society*, **62**, 113	N.L. 1	Norman Lockyer
Rimmer	*Memoirs Royal Astronomical Society*, **64**, 1	N.L. 2	,, ,,
Harper and Young	*Publ. Victoria Obs.*, **3**, No. 1	Vict.	Victoria
Adams, Joy, Humason	*Mount Wilson Cont.*, No. 319	M.W. 319	Mount Wilson

F to M Stars—continued

		Abbreviation	Observatory
Shapley and Lindblad	*Harvard Circulars* No. 228	H.C. 228	Harvard
H. Shapley and M. B. Shapley	*Harvard Circulars* No. 232	H.C. 232	,,
H. Shapley and A. Ames	*Harvard Circulars* No. 243	H.C. 243	,,
H. Shapley	*Harvard Circulars* No. 246	H.C. 246	,,

The results are given in Table I. M is the absolute magnitude; ΔM the correction to the absolute magnitude found according to the method described in the last column; n denotes the number of stars. The spectral classes at the head of the correction tables are always taken from the Henry Draper Catalogue.

The meanings of the abbreviations in the table are as follows:—

trig. = reduction by means of trigonometric parallaxes; method (*a*).
par. mot. = reduction by means of the parallactic motion; method (*b*).
τ = reduction by means of the τ component of proper motion; method (*c*).
$\pi_{m,\mu}$ = reduction by means of the mean parallaxes of stars of given proper motion and magnitude; method (*d*).
Cluster = reduction by means of the parallaxes of the Taurus and Ursa Major clusters; method (*e*).

The computations have been much facilitated by a card catalogue of spectroscopic absolute magnitudes kept at our laboratory. Each star is represented by a card on which the values of the absolute magnitude found by the different authorities are noted. The spectral class is taken from the Henry Draper Catalogue. The apparent magnitude corresponding to the absolute magnitude of spectroscopic binaries is the combined magnitude +0·5 in the case of stars showing two spectra, according to *Lick Bulletin*, No. 355, 1924.

TABLE I

Summary of Computed Corrections ΔM *to the Absolute Magnitude* M

M.W. 262. *B Stars*

M	Δ*M*	*n*	Method
−2·0	−0·5	15	M.S.
−1·8	−0·2	18	M.W. 147
−1·2	−0·2	16	M.S.
−0·6	+0·1	18	M.W. 147
−0·4	+0·3	13	M.S.

N.L. 3, *N.L.* 4, *N.L.* 6I. *B Stars*

M	Δ*M*	*n*	Method
−2·9	0·0	10	N.L. 3 with M.W. 147
−2·8	+0·3	22	N.L. 3 ,, N.L. 5
−2·3	+0·4	24	N.L. 3 ,, N.L. 5

N.L. 3, *N.L.* 4, *N.L.* 6I. *B Stars*—contd.

M	Δ*M*	*n*	Method
−1·9	+1·3	7	N.L. 6I with M.W. 147
−1·8	−0·1	20	N.L. 4 ,, N.L. 5
−1·7	+0·2	10	N.L. 3 ,, M.W. 147
−1·7	+0·2	29	N.L. 3 ,, N.L. 5
−1·6	+0·4	8	N.L. 4 ,, M.W. 147
−1·6	+0·4	18	N.L. 4 ,, N.L. 5
−1·6	0·0	13	N.L. 6I ,, N.L. 5
−1·4	+0·4	15	N.L. 6I ,, N.L. 5
−1·2	+0·5	16	N.L. 3 ,, N.L. 5
−1·2	+0·3	14	N.L. 6I ,, N.L. 5
−0·7	+0·6	16	N.L. 4 ,, N.L. 5

N.L. 5, N.L. 6II, N.L. 7. B Stars

M	Δ*M*	*n*	Method
−2·3	−0·1	14	N.L. 5 with M.W. 147
−1·9	+0·2	31	N.L. 5 ,, M.W. 262
−1·6	+0·2	14	N.L. 5 ,, M.W. 147
−1·3	+0·3	34	N.L. 5 ,, M.W. 147
−1·2	+0·3	10	N.L. 6II ,, M.S.
−1·0	+0·9	10	N.L. 6II ,, M.S.
−1·0	+0·5	9	N.L. 7 ,, M.W. 262
−0·9	+0·1	4	N.L. 6II ,, M.W. 147
−0·8	+0·4	6	N.L. 7 ,, M.W. 147
−0·7	+0·5	15	N.L. 5 ,, M.W. 147
−0·5	+0·4	27	N.L. 5 ,, M.W. 262
−0·1	+0·5	13	N.L. 7 ,, M.W. 262

M.S. B Stars

M	Δ*M*	*n*	Method
−2·2	−0·3	17	M.W. 147
−2·0	−0·3	10	M.W. 82
−1·9	−0·2	18	M.W. 147
−1·2	−0·2	28	M.W. 82
−0·5	+0·1	15	M.W. 147
−0·5	+0·2	14	M.W. 82

M.W. 244. A Stars

+0·8	−0·6	28	N.L. 8
+0·8	−0·6	20	Ye.
+0·9	−0·7	17	trig. π
+1·0	−0·6	8	Cluster
+1·1	−0·6	31	N.L. 8
+1·2	−0·9	233	$\pi_{m,\mu}$
+1·3	−0·6	21	Ye.
+1·4	−0·3	31	trig. π
+1·4	0·0	54	$\pi_{m,\mu}$
+1·6	−0·2	20	Ye.
+1·7	−0·3	24	N.L. 8
+1·7	−0·2	13	Cluster

Ye. A Stars

+0·7	−0·4	29	par. mot.
+0·8	−0·7	16	trig. π
+0·8	−0·2	32	$\pi_{m,\mu}$
+0·9	0·0	7	Cluster
+1·1	−0·7	7	trig. π
+1·1	−0·8	44	$\pi_{m,\mu}$
+1·2	−1·2	38	par. mot.
+1·5	−0·6	23	trig. π
+1·6	0·0	25	par. mot.
+1·6	−0·8	65	$\pi_{m,\mu}$
+1·8	+0·1	10	Cluster

N.L. 8. A Stars

M	Δ*M*	*n*	Method
+0·8	−0·5	32	par. mot.
+0·9	−0·6	39	$\pi_{m,\mu}$
+0·9	−0·6	34	trig. π
+0·9	0·0	7	Cluster
+1·1	−0·6	151	$\pi_{m,\mu}$
+1·1	−0·8	30	par. mot.
+1·2	0·0	67	$\pi_{m,\mu}$
+1·2	−0·6	46	trig. π
+1·4	−0·3	13	trig. π
+1·4	0·0	30	par. mot.
+1·4	−0·2	14	Cluster

Arc. 42. A Stars

+0·6	−0·6	14	trig. π
+0·7	−0·5	19	N.L. 8
+1·1	−0·8	13	N.L. 8
+1·2	+0·2	4	trig. π
+1·3	−0·3	6	Cluster
+1·6	−0·2	12	N.L. 8
+1·6	0·0	7	trig π

M.W. 199. F Stars

−1·2	−1·8	11	trig. π
−0·7	+0·4	14	par. mot.
+0·1	−1·3	35	$\pi_{m,\mu}$
+0·3	−0·5	39	τ
+1·0	−0·2	18	trig. π
+1·3	−0·7	17	par. mot.
+2·0	−0·7	61	$\pi_{m,\mu}$
+2·4	−0·2	17	trig. π
+3·2	−0·4	125	$\pi_{m,\mu}$
+3·3	0·0	36	trig. π
+3·8	+0·3	79	$\pi_{m,\mu}$
+4·0	0·0	25	trig. π

M.W. 199. G Stars

−1·8	−0·3	31	par. mot.
−1·3	−0·6	22	trig. π
−1·1	−0·1	41	$\pi_{m,\mu}$
−0·4	0·0	62	τ
+0·3	−0·3	124	$\pi_{m,\mu}$
+0·4	−1·3	25	par. mot.
+1·0	+0·1	66	$\pi_{m,\mu}$
+2·2	−0·2	29	trig. π
+3·1	−0·1	79	$\pi_{m,\mu}$
+4·2	−0·2	27	trig. π
+5·0	−0·5	147	$\pi_{m,\mu}$
+5·2	−0·3	32	trig. π

M.W. 199. *K Stars*

M	Δ*M*	*n*	Method
−0·2	−2·7	39	τ
−0·2	−1·9	28	trig. π
+0·1	−1·8	55	par. mot.
+0·3	−1·7	45	$\pi_{m,\mu}$
+0·3	−0·7	42	τ
+0·7	−0·4	96	trig. π
+0·8	−1·3	80	$\pi_{m,\mu}$
+0·9	−0·9	96	$\pi_{m,\mu}$
+1·0	−0·1	101	$\pi_{m,\mu}$
+2·2	−0·5	19	trig. π
+5·8	−0·1	20	trig. π
+6·5	−0·5	145	$\pi_{m,\mu}$

M.W. 319. *M Stars*

−1·1	−0·5	15	Vict.
−0·7	−0·4	9	trig. π
−0·7	+0·9	19	par. mot.
−0·4	−0·6	90	$\pi_{m,\mu}$
−0·3	−0·6	7	trig. π
−0·1	−0·2	95	$\pi_{m,\mu}$
−0·1	0·0	45	par. mot.
0·0	−0·3	34	Vict.
+0·1	−0·3	12	trig. π
+9·8	0·0	26	trig. π

N.L. 1. *F Stars*

+1·5	−0·6	6	M.W. 199
+1·8	−0·8	29	Vict.
+1·8	−0·4	33	trig. π
+1·9	−2·0	22	$\pi_{m,\mu}$
+2·4	−0·2	18	M.W. 199
+2·9	−0·4	82	$\pi_{m,\mu}$
+3·1	−0·1	24	Vict.
+3·1	0·0	30	trig. π
+3·2	−0·2	60	M.W. 199
+3·8	−0·2	11	$\pi_{m,\mu}$
+3·8	+0·1	29	trig. π
+3·9	+0·1	22	Vict.

N.L. 1. *G Stars*

−0·3	−0·3	11	M.W. 199
−0·1	−0·7	24	trig. π
+0·1	+0·2	22	Vict.
+0·2	−0·8	28	$\pi_{m,\mu}$
+0·5	−0·3	29	M.W. 199
+0·6	−0·2	14	$\pi_{m,\mu}$
+1·2	−0·2	7	M.W. 199
+2·4	−0·4	23	trig. π
+2·6	−0·1	33	$\pi_{m,\mu}$
+2·7	−0·2	19	Vict.
+2·9	−0·3	23	M.W. 199
+4·4	−0·2	24	M.W. 199
+4·6	−0·4	19	$\pi_{m,\mu}$
+4·8	−0·1	23	Vict.
+4·8	−0·1	23	trig. π

N.L. 1. *K Stars*

+0·3	−1·7	8	M.W. 199
+0·3	−1·1	51	Vict.
+0·3	−1·1	52	trig. π
+0·5	−1·4	39	M.W. 199
+0·6	−1·5	80	$\pi_{m,\mu}$
+0·8	−1·2	91	M.W. 199
+0·8	−0·1	106	$\pi_{m,\mu}$
+0·8	−0·8	52	Vict.
+0·8	−0·6	53	trig. π
+1·0	−1·2	49	M.W. 199
+2·1	+0·2	57	trig. π
+2·4	−0·6	48	Vict.
+2·6	−0·4	25	M.W. 199
+4·2	0·0	12	$\pi_{m,\mu}$

N.L. 1. *M Stars*

−0·9	+0·2	12	Vict.
0·0	−0·6	15	M.W. 319
0·0	−0·3	9	$\pi_{m,\mu}$
+0·1	−1·0	26	$\pi_{m,\mu}$
+0·2	−0·6	27	M.W. 319
+0·2	+0·1	10	$\pi_{m,\mu}$
+0·2	−0·3	16	Vict.

N.L. 2. *F Stars*

−1·1	−1·4	14	Vict.
−0·8	−0·4	14	M.W. 199
−0·2	−0·5	31	$\pi_{m,\mu}$
−0·2	−0·3	22	trig. π
+2·1	−0·7	28	$\pi_{m,\mu}$
+2·3	−0·3	15	Vict.
+2·5	−0·6	31	M.W. 199
+2·8	−0·4	67	$\pi_{m,\mu}$
+3·2	+0·2	23	trig. π
+3·3	+0·2	13	Vict.
+3·7	+0·1	9	M.W. 199

N.L. 2. *G Stars*

−2·6	−0·6	12	M.W. 199
−2·5	−1·4	14	Vict.
−1·9	+0·4	18	trig. π
−1·4	+0·5	36	$\pi_{m,\mu}$
−0·9	0·0	15	M.W. 199
−0·2	+0·3	23	$\pi_{m,\mu}$
+1·3	−0·2	17	$\pi_{m,\mu}$
+2·8	−0·2	27	$\pi_{m,\mu}$

N.L. 2. G Stars—contd.

M	ΔM	n	Method
+3·9	0·0	16	trig. π
+4·0	−0·1	12	Vict.
+4·4	−0·5	11	M.W. 199
+4·9	−0·4	5	$\pi_{m,\mu}$

N.L. 2. K Stars

−2·9	+0·5	17	trig. π
−1·7	−2·2	22	trig. π
−0·9	−0·6	15	Vict.
−0·5	−0·7	55	$\pi_{m,\mu}$
−0·3	−1·1	30	M.W. 199
−0·3	+0·5	20	trig. π
+0·2	−1·1	20	M.W. 199
+0·3	−0·7	65	$\pi_{m,\mu}$
+0·4	−0·6	12	Vict.
+0·7	−0·1	53	$\pi_{m,\mu}$
+0·8	−1·0	37	M.W. 199
+3·1	0·0	11	Vict.
+6·0	0·0	5	M.W. 199

N.L. 2. M Stars

−2·3	+0·3	8	M.W. 319
−2·2	0·0	12	Vict.
−1·4	+0·5	23	$\pi_{m,\mu}$
−1·1	+0·8	14	M.W. 319
−0·8	+0·4	10	M.W. 319
−0·8	+0·6	10	$\pi_{m,\mu}$
−0·7	+0·4	9	Vict.

Victoria. F Stars

−3·0	−1·9	13	trig. π
−0·5	+0·5	15	par. mot.
−0·2	+0·1	30	τ
0·0	−1·2	24	$\pi_{m,\mu}$
+1·0	−0·2	19	trig. π
+1·6	−0·3	30	par. mot.
+2·3	−0·5	93	$\pi_{m,\mu}$
+2·5	−0·2	24	trig. π
+3·3	−0·2	131	$\pi_{m,\mu}$
+3·3	−0·4	42	trig. π
+4·2	0·0	20	$\pi_{m,\mu}$
+4·3	−0·2	13	trig. π

Victoria. G Stars

M	ΔM	n	Method
	$m < 5·5$		
−0·8	−1·7	22	trig. π
0·0	−0·8	37	$\pi_{m,\mu}$
0·0	−1·3	31	τ
+1·3	−0·6	15	trig. π
+1·7	−0·5	67	$\pi_{m,\mu}$
+3·4	−0·1	23	$\pi_{m,\mu}$
+3·6	−0·5	10	trig. π
+4·7	−0·1	28	trig. π
+5·0	+0·1	6	$\pi_{m,\mu}$
	$m > 5·5$		
+0·3	+0·2	45	$\pi_{m,\mu}$
+2·1	+0·2	55	$\pi_{m,\mu}$
+3·2	−0·1	11	trig. π
+4·4	−0·2	38	$\pi_{m,\mu}$
+4·6	−0·2	27	trig. π

Victoria. K Stars

	$m < 5·5$		
−0·6	−0·6	24	par. mot.
−0·4	−1·2	32	τ
0·0	−1·0	54	$\pi_{m,\mu}$
+0·1	−0·9	45	trig. π
+0·3	−0·8	33	par. mot.
+0·8	−0·8	101	trig. π
+0·9	−0·2	64	$\pi_{m,\mu}$
+2·6	−0·6	15	trig. π
	$m > 5·5$		
0·0	+0·1	65	$\pi_{m,\mu}$
+5·6	0·0	22	$\pi_{m,\mu}$
+5·6	0·0	16	trig. π

Victoria. M Stars

−1·6	+0·5	30	trig. π
−1·3	+0·4	30	$\pi_{m,\mu}$
−1·1	+0·3	28	par. mot.
−1·0	+0·9	44	$\pi_{m,\mu}$
−0·6	+0·3	35	trig. π

4. *Reduction of Draper B Stars. N.L. 3 to N.L. 7; M.S. and M.W.* 262

The reduction of the B stars is difficult because the trigonometric parallaxes are too small to be of much use, and the mean parallaxes of stars of determined proper motion and apparent magnitude have not been computed for the B stars in *Gron. Publ.*, No. 34.

I do not want to use the parallactic motions on account of the tendency of the B stars to clustering. The systematic motions of the B stars in the region investigated by Kapteyn in *Mount Wilson Contributions*, No. 147, is undoubtedly different from the parallactic motion usually adopted. The Pleiades and the Perseus cluster have not been used for the reason explained above.

I have taken as the basis of the reduction of the absolute magnitudes of the B stars Kapteyn's values in *Mount Wilson Contributions*, Nos. 82 and 147. The only catalogue which can be directly compared with *Mount Wilson Contributions*, No. 82, is *Mount Stromlo*, Memoir No. 2, the other catalogues being restricted to the northern hemisphere whereas *Mount Wilson Contributions*, No. 82, is south. Several catalogues of B stars could be compared with *Mount Wilson Contributions*, No. 147. We have first reduced the catalogues which have a sufficient number of stars in common either with *Mount Wilson Contributions*, No. 82 or No. 147. We thus find a provisional system of absolute magnitudes of B stars by means of which the other catalogues have been reduced. In the case of a comparison with *Mount Wilson Contributions*, Nos. 82 and 147, therefore, the reduction of the system of absolute magnitudes is a direct one. In all other cases the reduction has been performed by means of the catalogue mentioned in the column "method." M.W. 262, for instance, has been reduced to the M.S. system, and the latter again has been corrected to the system adopted in this paper. For the Norman Lockyer (N.L.) values the first catalogue mentioned in the last column denotes the system of absolute magnitudes for which the correction is required, the second one either *Mount Wilson Contributions*, Nos. 82 or 147, with which it has been compared, or the intermediary catalogue by means of which it has been reduced to *Mount Wilson Contributions*, Nos. 82 and 147.

Of each catalogue reduced in this section we have used the stars called B in the Henry Draper Catalogue exclusively. The others have been omitted.

The basis of grouping is never the absolute magnitude for which the correction is required, but either another set of absolute magnitudes or the proper motion. In the case of the comparison of M.S. with *Mount Wilson Contributions*, No. 82, for instance, the stars are grouped according to the mean absolute magnitude of the authorities other than M.S., and the mean value of the absolute magnitudes of M.S. and *Mount Wilson Contributions*, No. 82, have been formed. The difference represents the correction to the M.S. absolute magnitude. The parallaxes given in *Mount Wilson Contributions*, No. 147, for each individual star really represent the mean parallaxes of stars of given value of the v-component of the proper motion towards the antapex. We have, therefore, used the v-component of proper motion as a basis of division, and formed the mean of Kapteyn's parallax values for each group. This represents the mean parallax of the group from which the mean absolute magnitude $\bar{M}$ is easily found. The difference between this $\bar{M}$ and the corresponding spectroscopic absolute magnitude represents the correction to the latter.

The absolute magnitudes of the catalogues N.L. 3, N.L. 4 and part of N.L. 6, called N.L. 6_I in this paper, depend on the estimates of line intensities, whereas the absolute magnitudes of N.L. 5, N.L. 6_{II} and N.L. 7 are based on the line character. In the latter case the absolute magnitudes of the three catalogues are on the same system because they have been derived from the same table printed in N.L. 5, p. 374.

In the case of N.L. 3, N.L. 4 and N.L. 6_I, the reduction curves used by Edwards were the same for N.L. 3 and N.L. 4.* The standard stars, on the other hand, did not yield the same measures for N.L. 6_I as for N.L. 3 and N.L. 4; but Edwards's reduction curves have been modified accordingly. Therefore the absolute magnitudes of N.L. 3, N.L. 4 and N.L. 6_I are probably on the same system, and a single reduction table has been adopted for the three of them.

5. *Reduction of Draper A Stars, N.L. 8, Ye., M.W. 244 and Arc. 42*

Only Draper A stars of the catalogues N.L. 8, Ye., M.W. 244 and Arc. 42 have been reduced in this section. The other objects of the catalogues mentioned, forming a small minority, have been omitted.

In the reduction of the N.L. 8 absolute magnitudes by means of the parallactic motion and the trigonometric parallaxes, the stars have been grouped according to the Ye. absolute magnitudes and *vice versa*. The adopted value of the solar motion in the reduction of the secular parallaxes is 16·0 km./sec.†

The column "method" of Table I denotes for M.W. 244 either the direct reduction by means of the $\pi_{m,\mu}$ trigonometric parallaxes and cluster parallaxes respectively or the intermediary catalogues by means of which the reduction has been performed. In the latter case the values of M.W. 244 are reduced to the system of N.L. 8 and Ye. respectively, and the latter again are corrected to the system of the present paper; the stars are grouped on the basis of the mean absolute magnitude of the catalogues *not* used in the reduction.

The stars of Arc. 42 have been reduced by means of the trigonometric parallaxes, the cluster parallaxes, and by means of N.L. 8 as an intermediary catalogue. The F stars of Arc. 42 have been omitted; the reduction by means of M.W. 199 and the trigonometric parallaxes yields the following comparison :—

Absolute Magnitude

Arc. 42	Present Paper	*M*
+2·1	+0·3	−1·8
+2·5	+2·2	−0·3
+2·9	+3·6	+0·7

The correction, therefore, changes very rapidly with the absolute magnitude of the F stars and I have thought it better not to use it.

* *M.N.*, 85, 439, 1925.
† *Gron. Publ.*, No. 37, p. 29, 1925.

6. *Reduction of Draper F to M Stars M.W.* 199, *M.W.* 319, *Vict., N.L.* 1, *N.L.* 2, *H.C.* 228, *H.C.* 232, *H.C.* 243 *and H.C.* 246

The corrections given in Table I for the F to M stars require little comment. In some cases the corrections have been derived by means of an intermediary catalogue given in the column "method"; see reduction of the A stars. The abbreviations used in this column are explained on p. 752.

The secular parallaxes are computed exclusively in the cases where the distribution of the proper motions of the stars with known spectroscopic absolute magnitude is the same as that given for the same spectral class, apparent magnitude and galactic latitude in *Gron. Publ.*, No. 30, Table XII. As the stars are always grouped on the basis of a set of absolute magnitudes different from the system for which the correction is required we can only use stars common to at least two catalogues. The Draper G stars of the Victoria Catalogue which have also been observed by another authority are not representative for the stars in general in point of proper motion; they show a preference for large values of the angular motion. The absolute magnitude of the Victoria G stars have therefore not been computed by means of the secular parallaxes.

The method of the $\pi_{m,\mu}$ has not been applied to the K and M stars for which the absolute magnitudes lie between $+2$ and $+4$ and $+2$ and $+7$ respectively. I think that the value of the correction found for these stars from the $\pi_{m,\mu}$ is probably systematically in error. The point is that the stars lie in the "gap" between giants and dwarfs, and therefore the frequency of occurrence is a low one. The $\pi_{m,\mu}$ for these stars have therefore been largely found by means of interpolation; the smoothing of the interpolation process has the tendency to form a group of stars of intermediate absolute brightness, whereas the true absolute brightness for most of the stars of the group is either that of the giants or that of the dwarfs.*

No corrections are given in Table I for the absolute magnitudes of the *Harvard Circulars*, Nos. 228, 232, 243, and 246. The values of *Harvard Circulars*, Nos. 228 and 232, are based on the system of *Mount Wilson Contributions*, No. 199; therefore the corrections given in Table IV for the latter catalogue can be applied. The reduction of *Harvard Circulars* Nos. 243 and 246 is somewhat difficult. These papers contain only stars in the southern hemisphere, where very few trigonometric parallaxes are available. The corrections for the absolute magnitudes which could be derived by the method of $\pi_{m,\mu}$ are in the final Table IV.

The corrections of the G and K stars of Victoria are given in Table I for two values of the apparent magnitude: brighter and fainter than the magnitude 5·5. The reason is that in my opinion these absolute magnitudes show an error, which is by no means negligible, depending on the apparent magnitude. It appears from the data of the table that the correction for the absolutely bright stars increases with decreasing apparent brightness. The

* This argument does not imply that stars of intermediate absolute brightness do not occur *at all*; it presupposes only that the frequency of stars of intermediate brightness is less than the maxima of giants and dwarfs.

conclusion is based largely on the $\pi_{m,\mu}$ by means of which the absolute brightness of giants, $m > 5{\cdot}5$, has been mainly derived. The difference between the corrections for apparently bright and faint stars might be due to an error in the $\pi_{m,\mu}$ of *Gron. Publ.*, No. 34. But the corrections to *Mount Wilson Contributions*, No. 199, derived from the $\pi_{m,\mu}$ do not show the dependence on the apparent magnitude, which is rather incomprehensible if the effect is due to an error in the latter.

TABLE II

Dependence of the Correction ΔM *on the Apparent Magnitude. Victoria Absolute Magnitudes*

Speciral Class G

M	*m*	Δ*M*	*n*	Method	Diff. in Δ*M*
+1·2	4·2	−0·4	9	trig. π	−0·6
+1·2	5·5	−1·0	9	,,	
+3·6	4·6	−0·5	10	,,	+0·4
+3·2	5·9	−0·1	11	,,	
+4·6	5·2	−0·1	28	,,	−0·1
+4·6	6·5	−0·2	27	,,	
−1·5	4·2	−0·6	6	$\pi_{m,\mu}$	0·0
−1·5	5·0	−0·6	5	,,	
−0·1	4·0	−1·1	12	,,	−0·2
−0·1	5·0	−1·3	15	,,	
+0·9	3·7	−1·4	12	,,	+1·0
+0·9	5·1	−0·4	16	,,	
+1·4	3·8	−0·7	21	,,	−0·2
+1·4	5·0	−0·9	18	,,	
−1·1	< 5·5	−0·8	11	,,	+1·0
−1·1	> 5·5	+0·2	13	,,	
+0·4	< 5·5	−0·8	26	,,	+1·2
+0·4	> 5·5	+0·4	32	,,	
+1·2	< 5·5	−0·5	26	,,	+0·9
+1·2	> 5·5	+0·4	22	,,	
+2·0	< 5·5	−0·5	41	,,	+0·8
+2·0	> 5·5	+0·3	33	,,	
+3·4	< 5·5	−0·1	23	,,	−0·1
+4·3	> 5·5	−0·2	35	,,	

I have investigated the error of the spectroscopic absolute magnitude depending on the apparent brightness for all authorities of Table I by computing the correction according to the methods described above for apparently

bright and faint stars. It appears that only the Victoria G and K stars are seriously affected by magnitude error. The magnitude error of the Victoria stars has been found from the $\pi_{m,\mu}$ as well as from the trigonometric parallaxes. In the case of the G stars I find the values of ΔM, the correction to the Victoria absolute magnitudes, for the values of M and m in the first and second columns (see Table II).

It appears from this table that the stars absolutely fainter than 3·0 show no error depending on the apparent magnitude, the correction ΔM being the same for apparently bright and faint stars.

For the giants, on the other hand, the value of ΔM for the group $m > 5{\cdot}5$ is in all cases appreciably larger than for the group $m < 5{\cdot}5$; for $m = 4{\cdot}0$ and 5·0 approximately the corrections do not differ. I therefore think that the magnitude error is negligible for the dwarfs, and that in the case of the giants it is shown only in the stars fainter than 5·5. Probably the spectra of the faintest stars observed at Victoria have been less dense than the spectra of the brighter stars, and this difference has influenced the estimate of the line intensities from which the absolute magnitude is derived.

In the case of the K stars the magnitude error changes more regularly with the apparent magnitude. Its value has been computed by means of the $\pi_{m,\mu}$ and the trigonometric parallaxes. Both methods show the magnitude error clearly and of approximately the same amount. The result is in the following table :

TABLE III

Correction to the Victoria Absolute Magnitudes of K Stars

m \ M	−0·4	0·0	+0·7	Mean
3·0	0·0	0·0	0·0	0·0
4·0	+0·1	+0·2	+0·2	+0·2
5·0	+0·6	+0·7	+0·8	+0·7
6·0	+1·5	+1·7	+1·7	+1·6

The correction has been taken arbitrarily equal to zero for $m = 3{\cdot}0$. The magnitude error is manifest for all three values of the absolute magnitude, −0·4, 0·0 and +0·7. The mean has been adopted for the giants. The dwarfs, as in the case of the G stars, show no magnitude error. We have adopted on the basis of the data of Table I the correction " zero " for the K giants, $m > 5{\cdot}5$. The correction given in Table I for K giants, $m < 5{\cdot}5$, holds for the mean magnitude 4·5 ; the value of the correction for the other apparent magnitudes has been found by means of Table III.

TABLE IV

Adopted Value of the Correction ΔM *to the Absolute Magnitude* M

(Extrapolated values are in parentheses)

B Stars

N.L. 3, N.L. 4, N.L. 6_I		N.L. 5, N.L. 6_{II}, N.L. 7		M.S.		M.W. 262	
M	Δ*M*	*M*	Δ*M*	*M*	Δ*M*	*M*	Δ*M*
< −3·0	(+0·3)	≦ −3·0	(−0·3)	< −2·2	(−0·2)	< −2·5	(−0·4)
−3·0 to −1·5	+0·3	−2·0	0·0	−2·2	−0·2	−2·5 to −2·1	(−0·3)
−1·0	+0·5	−1·0	+0·4	−1·2	−0·2	−2·0 ,, −1·6	−0·3
0·0	+0·5	0·0	+0·5	−1·1 to −1·0	−0·1	−1·5 ,, −1·2	−0·2
> 0·0	(+0·5)	> +1·0	(+0·5)	−0·9 ,, −0·8	0·0	−1·1 ,, −1·0	−0·1
				−0·7 ,, −0·6	+0·1	−0·9	0·0
				−0·5 ,, −0·4	+0·2	−0·8	+0·1
				> −0·4	(+0·3)	−0·7 to −0·6	+0·2
						−0·5 ,, −0·4	+0·3
						−0·3 ,, −0·1	(+0·3)
						> −0·1	(+0·4)

A Stars

N.L. 8		Ye.		M.W. 244		Arc. 42	
M	Δ*M*	*M*	Δ*M*	*M*	Δ*M*	*M*	Δ*M*
< +0·8	(−0·6)	< +0·7	(−0·4)	< +0·8	(−0·6)	< 0·0	(−0·5)
+0·8 to +1·1	−0·6	+0·7 to +1·8	−0·4	+0·8 to +1·2	−0·6	0·0	−0·5
+1·2 ,, +1·3	−0·3	> +1·8	(−0·4)	+1·3 ,, +1·4	−0·5	+1·0	−0·3
+1·4	0·0			+1·5 ,, +1·6	−0·4	+2·0	−0·1
> +1·4	(0·0)			+1·7	−0·3	> +2·0	(−0·1)
				> +1·7	(−0·3)		

F, G, K and M Stars

M.W. 199						M.W. 319	
F		G		K		M Giants	
M	Δ*M*	*M*	Δ*M*	*M*	Δ*M*	*M*	Δ*M*
≦ −2·0	(−1·3)	< −2·0	(−0·2)	≦ −1·0	(−2·0)	< −1·0	(−0·5)
−1·0	−1·3	−1·0	−0·2	0·0	−1·8	−1·0	−0·5
0·0	−1·0	0·0 to +3·0	−0·1	+1·0	−0·9	0·0	−0·2
+1·0	−0·6	+4·0	−0·2	+2·0	−0·5	> 0·0	(−0·2)
+2·0	−0·4	+5·0	−0·3	+3·0	−0·4		
+3·0	−0·1	> +5·0	(−0·3)	+4·0	−0·3	M Dwarfs	
+4·0	+0·1			+5·0	−0·3	all	0·0
≧ +5·0	(+0·1)			+6·0	−0·2		
				+7·0	(−0·1)		
				≧ +8·0	(0·0)		

Victoria

F		G				K					M Giants	
		$m < 5{\cdot}5$		$m > 5{\cdot}5$		$m \leqq 3{\cdot}0$		4·0	5·0	$\geqq 5{\cdot}5$		
M	ΔM	M	ΔM	M	ΔM	M	ΔM	ΔM	ΔM	ΔM	M	ΔM
$< -1{\cdot}0$	$(-0{\cdot}9)$	$\leqq -3{\cdot}0$	$(-1{\cdot}4)$	$\leqq -1{\cdot}0$	$(+0{\cdot}2)$	$\leqq -2{\cdot}0$	$(-1{\cdot}9)$	$(-1{\cdot}7)$	$(-1{\cdot}2)$	(0·0)	$\leqq -2{\cdot}0$	$(+0{\cdot}4$
$-1{\cdot}0$	$-0{\cdot}9$	$-2{\cdot}0$	$(-1{\cdot}2)$	0·0	$+0{\cdot}2$	$-1{\cdot}0$	$-1{\cdot}7$	$-1{\cdot}5$	$-1{\cdot}0$	(0·0)	$-1{\cdot}0$	$+0{\cdot}5$
0·0	$-0{\cdot}9$	$-1{\cdot}0$	$-1{\cdot}2$	$+1{\cdot}0$	$+0{\cdot}2$	0·0	$-1{\cdot}4$	$-1{\cdot}2$	$-0{\cdot}7$	0·0	$\geqq$ 0·0	$(+0{\cdot}6$
$+1{\cdot}0$	$-0{\cdot}7$	0·0	$-1{\cdot}0$	$+2{\cdot}0$	0·0	$+1{\cdot}0$	$-1{\cdot}1$	$-0{\cdot}9$	$-0{\cdot}4$	(0·0)		
$+2{\cdot}0$	$-0{\cdot}5$	$+1{\cdot}0$	$-0{\cdot}6$	$+3{\cdot}0$	0·0	$+2{\cdot}0$	$-0{\cdot}7$	$-0{\cdot}6$	$-0{\cdot}4$	(0·0)		
$+3{\cdot}0$	$-0{\cdot}2$	$+2{\cdot}0$	$-0{\cdot}3$	$+4{\cdot}0$	$-0{\cdot}1$	$+3{\cdot}0$	$-0{\cdot}3$	$-0{\cdot}3$	$-0{\cdot}3$	(0·0)		
$+4{\cdot}0$	$-0{\cdot}2$	$+3{\cdot}0$	$-0{\cdot}2$	$\geqq +5{\cdot}0$	$(-0{\cdot}1)$	$+4{\cdot}0$	$(-0{\cdot}1)$	$(-0{\cdot}1)$	$(-0{\cdot}1)$	(0·0)		
$\geqq +5{\cdot}0$	(0·0)	$+4{\cdot}0$	$-0{\cdot}1$			$+5{\cdot}0$	(0·0)	(0·0)	(0·0)	0·0		
		$\geqq +5{\cdot}0$	(0·0)			$> +5{\cdot}0$	(0·0)	(0·0)	(0·0)	(0·0)		

N.L. 1

F		G		K		M Giants	
M	ΔM	M	ΔM	M	ΔM	M	ΔM
$< +1{\cdot}0$	$(-1{\cdot}1)$	$< 0{\cdot}0$	$(-0{\cdot}3)$	$< 0{\cdot}0$	$(-1{\cdot}1)$	$< -0{\cdot}5$	$(-0{\cdot}4)$
$+1{\cdot}0$	$-1{\cdot}1$	0·0 to +5·0	$-0{\cdot}3$	0·0	$-1{\cdot}1$	$-0{\cdot}5$ to $+0{\cdot}5$	$-0{\cdot}4$
$+2{\cdot}0$	$-0{\cdot}9$	$> +5{\cdot}0$	$(-0{\cdot}3)$	$+1{\cdot}0$	$-0{\cdot}8$	$> +0{\cdot}5$	$(-0{\cdot}4)$
$+3{\cdot}0$	$-0{\cdot}2$			$+2{\cdot}0$	$-0{\cdot}2$		
$+4{\cdot}0$	0·0			$+3{\cdot}0$	$(-0{\cdot}1)$		
$> +4{\cdot}0$	(0·0)			$+4{\cdot}0$	0·0		
				$> +4{\cdot}0$	(0·0)		

N.L. 2

F		G		K		M Giants	
M	ΔM	M	ΔM	ΔM	ΔM	M	ΔM
$< -1{\cdot}0$	$(-0{\cdot}8)$	$\leqq -3{\cdot}0$	$(-1{\cdot}0)$	$< -2{\cdot}0$	$(-0{\cdot}8)$	$< -2{\cdot}0$	$(+0{\cdot}4)$
$-1{\cdot}0$	$-0{\cdot}8$	$-2{\cdot}0$	$-0{\cdot}3$	$-2{\cdot}0$ to 0·0	$-0{\cdot}8$	$-2{\cdot}0$ to $-0{\cdot}5$	$+0{\cdot}4$
0·0	$-0{\cdot}5$	$-1{\cdot}0$	$+0{\cdot}3$	$+1{\cdot}0$	$-0{\cdot}2$	$> -0{\cdot}5$	$(+0{\cdot}4)$
$+1{\cdot}0$	$-0{\cdot}4$	0·0	$+0{\cdot}1$	$+2{\cdot}0$ to $+6{\cdot}0$	0·0		
$+2{\cdot}0$	$-0{\cdot}3$	$+1{\cdot}0$ to $+4{\cdot}0$	$-0{\cdot}2$	$> +6{\cdot}0$	(0·0)		
$+3{\cdot}0$	0·0	$> +4{\cdot}0$	$(-0{\cdot}2)$				
$+4{\cdot}0$	0·0						
$> +4{\cdot}0$	(0·0)						

N.L. 2—continued

H.C. 243. K Stars	
M	Δ*M*
+0·3	−1·0
+0·7	−0·3

H.C. 246. K Stars	
M	Δ*M*
−0·3	−1·5
+0·4	−0·7

H.C. 246. G Stars	
M	Δ*M*
−0·2	−0·5
+0·4	0·0
+3·2	0·0

7. *Table of Adopted Corrections. Discussion of the Results*

The adopted corrections are given in Table IV. *M* is the spectroscopic absolute magnitude, Δ*M* is the correction. The spectral classes for which the corrections are valid are derived from the Henry Draper Catalogue. The adopted corrections are the means of the values of Table I. The weights have been roughly taken into account. Extrapolated values are in parentheses.

It appears from Table IV that in most cases the corrections to be applied to the absolutely bright stars are negative. This may be partly explained by the fact that most authorities use the measured line intensities or estimated line character as a basis of grouping, the mean computed absolute magnitude of the group being taken to correspond with its mean line intensity. It is clear that some of the stars selected on the basis of an extreme value of the line intensity on the absolutely bright side will not be bright at all but simply fall into the bright group by the accidental errors. Thus the absolute magnitude corresponding to the extreme value of the line intensity will be found fainter than in the present paper, where the stars have always been grouped on the basis of the line intensities of an authority *other* than that for which the correction is required.

This does not hold for the Victoria results, because Young and Harper have used as a basis of grouping the mean of the Mount Wilson absolute magnitudes and those derived from trigonometric parallaxes.* But it is stated on p. 35 of the quoted publication that the Victoria absolute magnitudes of the absolutely bright stars are probably too faint. In my opinion there can be no doubt that the large negative corrections applied to the giant stars of type F to K in Table IV are required by the system adopted in the present paper. Both the $\pi_{m,\mu}$ and the trigonometric parallaxes give approximately the same value for the correction. The system adopted may therefore be questioned, but not the corrections, the system being once accepted. Relatively to the latter I can only say that the mean parallaxes $\bar{\pi}_{m,\mu}$ of *Gron. Publ.*, No. 34, are in substantial agreement with the Allegheny parallaxes and differ very little from the McCormick values.†

The parallactic motion and the τ-component of proper motion have been

* Young and Harper, *Publications of the Victoria Observatory*, **3**, No. 1, p. 33, 1924.

† *Gron. Publ.*, No. 37, p. 5, 1925.

used to check the large negative corrections applied in Table IV to the giant F to K stars. It appears that the corrections derived by these methods agree with the values finally adopted on the average within a few tenths of a magnitude.

8. *Distribution of Spectroscopic Absolute Magnitudes of Stars of given True Absolute Magnitude. F to M Stars*

The form of the distribution curve of the spectroscopic absolute magnitude M_s for stars of the same true absolute magnitude M depends on the errors of observation of the line intensities as well as on the so-called cosmic errors of the spectroscopic magnitudes (see p. 745). We shall investigate the form of this curve produced by both kinds of error together. The error in the spectroscopic magnitude $\Delta M_s = M_s - M$. Therefore the distribution curve of the M_s for stars of given M is easily found if the distribution curve of the ΔM_s has been derived from the data of observation.

(a) *F to M Stars.*—The distribution curve of the ΔM_s has been investigated for the dwarf F to M stars by means of the trigonometric parallaxes.

The stars have been selected on the basis of their proper motion exceeding $0''{\cdot}500$ per annum.* The absolute magnitude derived from the trigonometric parallax M_t is compared with the spectroscopic absolute magnitude M_s of M.W. 199, corrected according to Table IV of the present paper. Only long-focus photographic parallax determinations have been taken into consideration; stars with a probable error of the parallax exceeding $\pm 0''{\cdot}008$ have been omitted. The difference $M_t - M_s$ is equal to the differences of the errors in the trigonometric and spectroscopic absolute magnitudes $\Delta M_t - \Delta M_s$. The distribution curve of these differences depends on the frequency curves of the ΔM_t and the curve of the ΔM_s.

Let

$f(\Delta M_s)d(\Delta M_s)$ be the frequency distribution of ΔM_s;
$\phi(\Delta M_t)d(\Delta M_t)$,, ,, ,, ΔM_t;
$\psi(x)dx$,, ,, ,, $x = \Delta M_t - \Delta M_s$;

then

$$\psi(x) = \int_{-\infty}^{+\infty} f(\Delta M_s)\phi(x + \Delta M_s)d(\Delta M_s). \qquad (7)$$

The unknown function $f(\Delta M_s)$ can be solved from this equation as soon as ϕ has been found, the curve ψ being known from the counted numbers $M_t - M_s$.

The function $\phi(\Delta M_t)d(\Delta M_t)$ is derived from the Gaussian distribution of the errors $\Delta\pi_t$ in the trigonometric parallaxes.

The errors ΔM_t and $\Delta\pi_t$ are connected by the formula

$$\Delta\pi_t = \pi\{10^{0.2(\Delta M_t)} - 1\}, \qquad (8)$$

where π is the true parallax.

* Luyten, *Lick Bulletin*, No. 344, 1923.

Therefore the quantities $\pi\{10^{0.2(\Delta M_t)} - 1\}$ are distributed in accordance with a Gaussian curve, the probable deviation of which is equal to the probable error of the trigonometric parallaxes used. The distribution curve of the ΔM_t therefore depends on the parallax of the stars ; we have computed the frequency of the ΔM_t for a number of equidistant values of π, and the average of these frequency curves is taken with weights equal to the number of stars in each parallax interval, from which the unknown function f is derived. This average frequency curve represents the distribution of the ΔM_t of the stars, $\mu > 0''{\cdot}500$, used in this section.

The probable error of the trigonometric parallaxes is larger than the value given by the parallax authorities. It has been taken equal to $\sqrt{r^2 + 0''{\cdot}007^2}$ in accordance with *Gron. Publ.*, No. 34, p. 5, 1923. The average probable error of the stars used is found to be $\pm\ 0''{\cdot}007$, the average number of independent parallax determinations of a single star being 2·4.

Let us assume that the function f is a Gaussian curve with a probable deviation 0·32, and compute the function $\psi(x)$ numerically according to the formula (7). We then find the frequencies of Table V.

TABLE V

Observed Distribution Curve of $x = \Delta M_t - \Delta M_s$ *compared with the Theoretical Curve of Formula* (7)

x	Observed	Computed	O − C
< −4·05	4	0	−4
−4·05 to −3·05	1	1	0
−3·05 ,, −2·05	2	2	0
−2·05 ,, −1·05	11	11	0
−1·05 ,, −0·65	9	17	+8
−0·65 ,, −0·45	19	14	−5
−0·45 ,, −0·25	18	18	0
−0·25 ,, −0·05	25	22	−3
−0·05 ,, +0·15	18	22	+4
+0·15 ,, +0·35	20	21	+1
+0·35 ,, +0·55	23	17	−6
+0·55 ,, +0·95	13	21	+8
+0·95 ,, +1·95	12	11	−1
> +1·95	3	1	−2

The computed numbers are found by multiplying the frequencies of the formula (7) by 178, the total numbers of stars. Although the residuals are in some cases pretty large, they do not have a systematic character except that the computed numbers for very small and very large values of x are too small. By taking a larger value for the probable deviation of the curve $f(\Delta M_s)$ the residuals for the extreme ends of the curve will be diminished. But I do not think it would be wise to do this. In most cases the large values of x, either negative or positive, are probably due to large errors *not* in the spectroscopic absolute magnitude but in the trigonometric values, and it is therefore not advisable to give these values of x a great weight in the determination of the frequency function $f(\Delta M)_s$. This conclusion is based on the fact that for

the stars with values of $x < -4{\cdot}05$ and $> +1{\cdot}95$ the trigonometric parallax values found by different authorities are rather discordant and that at least one of the parallax determinations for each of the stars with $x < -4{\cdot}05$ is negative. I therefore think that the observed frequencies of the x are satisfactorily represented by the assumption that the spectroscopic absolute magnitudes of stars of a given true absolute magnitude are distributed in accordance with a Gaussian curve.

It must be noted that the distribution curve of the x is mainly determined by the distribution of the ΔM_s and that the ΔM_t have only a secondary influence. If it were otherwise it would have been impossible to find the distribution curve of the ΔM_s from that of the x.

(b) *F to M Giants.*—It is much more difficult to find the distribution curve of the spectroscopic absolute magnitudes of stars of the same true absolute magnitude for the absolutely bright stars than to find it for the faint ones. Trigonometric parallaxes are of no use because they are too small. I have tried to find the curve by means of double stars for which both components have been observed; the difference of the errors in the spectroscopic absolute magnitudes of both components is equal to the difference in the observed spectroscopic absolute magnitudes minus the difference of the apparent magnitudes. But only 4 double stars for which both components are brighter than $M = 1{\cdot}0$ are available.

The differences of the absolute magnitudes found by different authorities for the same star have also been used. I have tried the F and G stars brighter than $M = 1{\cdot}0$, observed both by the Victoria and by the Mount Wilson astronomers. The differences are indeed distributed in accordance with a Gaussian curve with a probable deviation of 0·45. But this is not a decisive test because the difference will contain only a fraction of the cosmic error, this error being at least partly the same for both the Victoria and the Mount Wilson observations. However this may be, I have assumed that the Gaussian character of the distribution curve of the ΔM_s holds also in the case of the F to M giants.

9. The probable deviations of the Gaussian curve of the ΔM_s for F to M stars have been found as follows:—

(a) *F to M Giant Stars.*—The spread of the differences $\pi_t - \pi_s$ between the trigonometric and the spectroscopic parallaxes depends on the probable error of the trigonometric parallax as well as on the probable error of the spectroscopic absolute magnitude from which the parallax is derived. The probable error of the spectroscopic absolute magnitude has been found from the differences $\pi_t - \pi_s$ in the following way:—

$\Delta\pi_t$ is the observational error in the trigonometric parallax;
$\Delta\pi_s$ is the error in spectroscopic parallax including the cosmic error.

We have

$$\overline{(\pi_t - \pi_s)^2} = \overline{(\Delta\pi_t - \Delta\pi_s)^2} = \overline{(\Delta\pi_t)^2} + \overline{(\Delta\pi_s)^2} = \epsilon^2 + \overline{(\Delta\pi_s)^2}, \qquad (9)$$

where a bar denotes the mean value of the quantity concerned and ϵ is the mean error of the trigonometric parallaxes.

The value of $\overline{(\pi_t - \pi_s)^2}$ can be easily found by a comparison of the trigonometric and spectroscopic parallaxes. Therefore the value of $\overline{(\Delta\pi_s)^2}$ can be derived from the formula (9). If we assume that the errors $\Delta \log \pi_s$ in the logarithms of the spectroscopic parallaxes are distributed in accordance with a Gaussian curve with a modulus of precision h, the value of this modulus is easily found from $\overline{(\Delta\pi_s)^2}$ as follows:—

The distribution curve of $\Delta \log \pi_s$ is

$$\frac{h}{\sqrt{3\cdot141}} e^{-h^2(\Delta \log \pi_s)^2} d(\Delta \log \pi_s). \tag{10}$$

Further,

$$\Delta \log \pi_s = \log\left(\frac{\Delta\pi_s}{\pi} + 1\right) \tag{11}$$

if π is the true parallax.

Substitution of (11) in (10) gives the distribution curve of the $\Delta\pi_s$ from which the mean value of $(\Delta\pi_s)^2$ for all values of $\Delta\pi_s$ is easily found. The result is :

$$\overline{(\Delta\pi_s)^2} = \pi^2\left\{e^{\frac{1}{\mathrm{mod}^2 h^2}} - 2e^{\frac{1}{4\,\mathrm{mod}^2 h^2}} + 1\right\}. \tag{12}$$

This value holds for stars of true parallax π. For objects at different distances :

$$\overline{(\Delta\pi_s)^2} = \overline{\pi^2}\left\{e^{\frac{1}{\mathrm{mod}^2 h^2}} - 2e^{\frac{1}{4\,\mathrm{mod}^2 h^2}} + 1\right\}, \tag{13}$$

where $\overline{\pi^2}$ is the mean square of the parallaxes used in the determination of h.

Finally, h is found by means of the formulæ (9) and (13), the mean error of the trigonometric parallaxes, ϵ, being assumed to be known. The method has first been applied to the Victoria F to M stars. We used only objects which fulfil the following conditions :—

the Mount Wilson absolute magnitude is $< 0\cdot5$;
the annual proper motion is $< 0''\cdot200$;
the apparent magnitude is $< 3\cdot5$;
the probable error of the adopted trigonometric parallax is $< \pm 0''\cdot008$.

The first and second conditions were imposed in order to get giant stars, the third one in order to avoid very small values of the parallax which might give difficulties in the determination of the distribution of the true parallaxes, needed for the derivation of $\overline{\pi^2}$. The spectroscopic parallax has been computed from the absolute magnitude corrected according to Table IV of the present paper. The trigonometric parallax system has been discussed above. The mean square $\pi_t - \pi_s$ is easily found, and formula (9) yields $\overline{(\Delta\pi_s)^2}$. The mean error of the trigonometric parallax has been found by means of the rules given by Schlesinger.* The result is $h = 7\cdot20$. Therefore the modulus

* *General Catalogue of Stellar Parallaxes*, 1924.

of precision of the Gaussian curve of the error in M_s is 1·44, and the corresponding probable error $= \pm 0{\cdot}33$.

A similar computation for the spectroscopic absolute magnitudes of giant stars of *Mount Wilson Contr.*, Nos. 199 and 319, yields $r = \pm 0{\cdot}66$.

We thus find a larger error for the Mount Wilson absolute magnitudes than for the Victoria values. There are some indications that this difference is real in the case of the K giant stars (see the end of this section).

The mean value for both series is $r = \pm 0{\cdot}50$.

(b) *F to M Giants and Dwarfs.*—The probable deviation r of the curve of the ΔM_s has been found in the second place by means of a comparison of the absolute magnitudes of *Mount Wilson Contr.*, No. 199, and Victoria. I find

$$\left.\begin{aligned} r &= \pm 0{\cdot}22 \text{ for stars with } M > 3{\cdot}0 \\ r &= \pm 0{\cdot}45 \quad ,, \quad ,, \quad < 1{\cdot}0 \end{aligned}\right\}. \tag{14}$$

The values are really lower limits because the cosmic errors, being partly the same for both observations, will be eliminated from the difference.

(c) *F to M Giants and Dwarfs.*—Consider a double star for which the absolute magnitudes of both components have been observed. The difference of the absolute magnitudes minus the difference of the apparent magnitudes is equal to the difference of the errors in the spectroscopic magnitudes. This leads to a determination of r. We find

$$\begin{aligned} r &= \pm 0{\cdot}28 \text{ from about 60 stars with } M > 2 \\ &= \pm 0{\cdot}45 \text{ from 4 stars with } M < 1{\cdot}0. \end{aligned}$$

(d) *F to M Dwarfs.*—The comparison of spectroscopic and trigonometric absolute magnitudes discussed in section 8 gives $r = \pm 0{\cdot}32$ for stars with $M > 2$.

(*e*) The following values have been found by other authorities :—

Hertzsprung, from double stars, $r = \pm 0{\cdot}45$ for dwarfs.*
Adams, Joy, Strömberg, Burwell, from comparison with trigonometric parallaxes, $r = \pm 0{\cdot}34$ for dwarfs.†
Young and Harper, from comparison with trigonometric parallaxes, $r = \pm 0{\cdot}4$ for dwarfs.‡

I have assumed the following values for the F to M stars :—

$$\begin{aligned} r &= \pm 0{\cdot}30 \text{ for absolute magnitudes } > 3{\cdot}0 \\ &= \pm 0{\cdot}42 \quad ,, \quad ,, \quad ,, \quad 1{\cdot}0 \text{ to } 3{\cdot}0 \\ &= \pm 0{\cdot}55 \quad ,, \quad ,, \quad ,, \quad < 1{\cdot}0 \end{aligned}$$

In *Gron. Publ.*, No. 34, pp. 60 to 68, I have discussed the Mount Wilson spectroscopic magnitudes. It was found there that the spectroscopic criteria of absolute brightness are less sensitive for absolutely bright stars than for absolutely faint ones, the change of the line intensity per unit of absolute magnitude being considerably smaller for giants than for dwarfs. This

* *Astronomische Nachrichten*, **208**, 265, 1918.
† *Mount Wilson Contributions*, No. 199, p. 15, 1921.
‡ *Victoria Publications*, **3**, No. 1, p. 40, 1924.

result agrees with the probable errors for luminous and faint stars found in the present paper. The distribution curve of the spectroscopic absolute magnitudes for stars of given apparent magnitude points in the same direction. This curve shows in some cases a very sharp maximum, especially for the Mount Wilson K stars. The number of stars in some of the absolute magnitude groups, if corrected for the errors of observation, is negative, which, of course, is impossible. The explanation probably is that too large a number of stars have been classified in the absolute magnitude group of the maximum frequency, exactly as would have been the case if the spectroscopic criteria of the K giants were not sufficiently sensitive to differences in absolute magnitude. If this explanation is correct, no trustworthy absolute magnitudes can be found in these cases from the observed line intensities.

10. *Distribution of Spectroscopic Absolute Magnitudes of Stars of given True Absolute Magnitude. A Stars*

The form of the frequency curve of the ΔM_s is found by a comparison of the spectroscopic absolute magnitudes with the values given by Rasmuson * for the Taurus and Ursa Major clusters. Only the stars enumerated on p. 750 have been used. The stars of M.W. 244 and Ye. have been investigated. The results are given in Table VI. The columns headed "Obs." give the observed numbers of differences ΔM between Rasmuson's absolute magnitudes and the spectroscopic values ; the columns headed "Comp." give the numbers computed with a Gaussian distribution. The probable errors are $\pm$ 0·35 for the Ye. magnitudes and $\pm$ 0·37 for the values of M.W. 244. The agreement is satisfactory.

TABLE VI

Distribution of the Differences ΔM *between Rasmuson's Absolute Magnitudes of the Taurus and Ursa Major Clusters and the Spectroscopic Magnitudes*

M	Ye.			M.W. 244		
	Obs.	Comp.	O − C	Obs.	Comp.	O − C
< −1·35	0	0	0	0	0	0
−1·35 to −1·15	0	0	0	0	1	−1
−1·15 „ −0·85	1	1	0	2	2	0
−0·85 „ −0·55	2	2	0	8	6	+2
−0·55 „ −0·25	4	4	0	5	9	−4
−0·25 „ +0·05	6	5	+1	14	12	+2
+0·05 „ +0·35	2	5	−3	11	11	0
+0·35 „ +0·65	6	3	+3	9	8	+1
+0·65 „ +0·95	0	1	−1	5	4	+1
+0·95 „ +1·25	1	1	0	2	2	0
> +1·25	0	0	0	0	1	−1

* *Meddelanden från Lunds Astronomiska Observatorium*, série II, Nr. 25, 1923.

The probable error of Rasmuson's absolute magnitudes depends on the accidental error of the proper motion from which the absolute magnitude is derived and on the probable value of the internal motion of the cluster. For the latter we take 0″·0075 in the case of the Hyades, in accordance with the value found in *Gron. Publ.*, No. 35, p. 11, 1924. The linear internal motion for the Taurus cluster therefore is

$$4{\cdot}74 \times \frac{0''{\cdot}0075}{0{\cdot}0270} = 1{\cdot}3 \text{ km./sec.}$$

The same value is taken to hold for the Ursa Major cluster. We find that the probable error of Rasmuson's absolute magnitudes is ± 0·2. It follows that the probable error of the spectroscopic absolute magnitudes is

$$\sqrt{(0{\cdot}36)^2 - (0{\cdot}20)^2} = \pm\, 0{\cdot}30.$$

The value for this probable error has further been found by means of double stars of which both components have been observed for absolute magnitude. The result is $r = \pm\, 0{\cdot}23$ for 7 Yerkes stars and ± 0·24 for 5 Mount Wilson stars. The differences between the Yerkes and Mount Wilson magnitudes for the same stars yield $r = \pm\, 0{\cdot}23$. This is a lower limit for the reason explained above.

We adopt $r = \pm\, 0{\cdot}30$.

11. *Probable Deviation of the Gaussian Curve of the* ΔM_s *for the B Stars*

In the case of the B stars the Gaussian character of the frequency curve of the ΔM_s has been assumed and the probable deviation of this curve has been determined as follows :—

The comparison of the difference in spectroscopic magnitude of the components of double stars with the difference in apparent magnitude gives $r = \pm\, 0{\cdot}60$ (8 double stars).

The comparison of the spectroscopic absolute magnitudes with Kapteyn's values in *Mount Wilson Contributions*, Nos. 82 and 147, gives $r = \pm\, 0{\cdot}42$ (52 stars) and $r = \pm\, 0{\cdot}43$ (33 stars) respectively. The probable errors of Kapteyn's absolute magnitudes have been found from the probable errors of the parallaxes given in his papers.

The comparison of the spectroscopic absolute magnitudes found by different authorities for the same stars yields $r = \pm\, 0{\cdot}25$. As explained above, this represents a lower limit. I have adopted $r = \pm\, 0{\cdot}42$.

12. The probable deviations of the Gaussian distributions of spectroscopic absolute magnitudes for stars of given true absolute magnitude are summarised in the following table:—

TABLE VII

Probable Deviations r of the Gaussian Distributions of Spectroscopic Absolute Magnitudes for Stars of given True Absolute Magnitude

Spectral Class	Absolute Magnitude	r
B	all	± 0·42
A	all	± 0·30
F to M	> 3·0	± 0·30
F ,, M	1·0 to 3·0	± 0·42
F ,, M	< 1·0	± 0·55

My thanks are due to Mr. J. B. van Wijk and Mr. T. Schat for their efficient help in the computations.

THE RADIAL VELOCITIES OF 343 STARS.

G. Shajn and V. Albitzky.

The present paper gives an abstract of the results of radial velocity observations secured during 1929–31 at the Simeis branch of the Pulkovo Observatory with the 40-inch reflector made by Sir Howard Grubb & Co. A detailed account of the work will shortly appear in the publications of the Pulkovo Observatory.

The spectrograph, used at the Cassegrain focus, is that of the Société Genevoise d'Instruments de Physique, with optical parts by Zeiss. The prism is of Jena glass, catalogue type 0·118″, and the camera objective is of Zeiss chromatic type (focal length 550 mm.), giving a satisfactory definition from λ 3600 to $H\alpha$, the difference in focus being about 0·25 mm. The linear dispersion is about 36 A. per millimetre at $H\gamma$. With the dispersion used, in good atmospheric conditions an hour's exposure gives the spectrum of a star of magnitude 6·3.

For the reductions of spectrograms two systems of wave-lengths have been used for the early- and late-type stars separately. The former is that compiled at Victoria.* For G, K and M stars we have used the lines and wave-lengths from the Mount Wilson Catalogue of 1013 stars.† A correction was added to reduce the wave-lengths in both systems to the international system.

The programme includes 301 stars mostly between magnitudes 5·75 and 6·50, and, in addition, 42 standard and other stars, with well-known velocities, down to magnitude 4·9 from the Lick Catalogue.‡ The main observed material is far from homogeneous as regards spectral type, but contains rather a selection of stars whose radial velocities have not hitherto been observed. Among the stars of Table I the percentages of types B8–A5, F and G–K are 34, 45 and 21 respectively. The inclusion of 135 stars of

* *Pub. D.A.O., Victoria*, **2**, No. 1, 1921.
† *Ap. J.*, **57**, 149, 1923.
‡ *Pub. Lick Obs.*, **16**, 1928.

type F in List 1 increases by about 15–20 per cent. the number of published velocities of this class. The spectrograms of the stars of types B, A and F may be of interest for the future investigation of the diffuseness, width, depth and other characteristics of spectrum lines, which for the stars even of the same spectral sub-class are highly various.

A minor detail of the programme is the attention paid to the radial velocities of double stars. We think that in three cases—βGC 1939, 6872 and 8788—the common radial velocity for both components may be considered as probably real.

Among the stars of List 1 there is a considerable number of new spectroscopic binaries: 15 B8–A5, 14 Fo–F5 and 1 G5.

A large radial velocity was found for HR 1908.

In order to derive and check the systematic error we have obtained the radial velocities of the standard and other brighter stars. An investigation of the systematic error on the basis of the study of the stars with known velocities shows that a correction $-0{\cdot}5$ km./sec. is to be added to the velocities of stars of types G, K, M and $-0{\cdot}1$ km./sec. to those of the B8–F8 stars.

The accuracy of the observations depends largely on the quality of the spectra. In the case of G–M and A–F stars with numerous and good lines the probable error of the velocity is about $\pm$ 1·2 km./sec., and that for a single plate is $\pm$ 2·4 km./sec.; for the early-type stars with few good or many fair lines the probable errors of a velocity and of a plate are $\pm$ 1·5 and $\pm$ 3·0 respectively; for the spectra with many poor lines the corresponding errors are $\pm$ 2·4 and $\pm$ 5·1; and, lastly, for the spectra with few and poor lines, $\pm$ 3·5 and $\pm$ 7·0 km./sec. respectively.

A by-product is the investigation of the system of wave-lengths of the lines used at Mount Wilson (mostly for late-type stars) and that for type A–F elaborated at Victoria. This is carried out on the basis of the residuals of radial velocities for individual lines on all good spectrograms. For many lines there are found sensible residuals which are primarily due to the errors of the assumed wave-lengths of the lines in question, and partly to the difference in the constants of the spectrographs (resolving power, purity). For several lines there is found a variation of wave-length with spectral type between the limits A–F and G–M. A detailed investigation of the problem in question and also of the systematic error and its dependence on the instrumental and other factors will be given in a Bulletin of the Pulkovo Observatory.

In Tables I and II, which are self-explanatory, are given the radial velocities of faint stars (List 1) and brighter ones (List 2). On the average, four spectrograms for each star were obtained. Many spectrograms were measured twice. With regard to List 1, V. Albitzky (A) is responsible for 118 stars and 521 spectrograms, G. Shajn (S) for 185 stars and 798 spectrograms, and other observers for 14 spectrograms.

The measurers are: V. Albitzky, 497; Mrs. P. Shajn, 1299; and G. Shajn, 499 spectrograms. In addition, Mrs. M. N. Abramova and Mrs. F. M. Neujmina have partly shared in the computations.

TABLE I

(*List* 1)

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates	P.E.	Obs.	
	h m	° ′							
17	0 3·5	+36 4	6·14	F5	− 14·0	4	± 1·0	A	
40	8·2	+26 25	6·30	F5	− 13·0	4	1·8	A	
50	9·8	+ 8 16	5·87	F5	...	4	...	A	Sp. Bin. Two spectra. Separation 186 km.
59	11·5	+ 7 41	6·19	G5	+ 3·7	3	3·0	A	
71	13·4	+30 58	5·80	Ao	− 4·2	4	1·2	A	
81	15·8	+10 25	6·55	Ao	− 17·8	4	2·3	A	
131	27·3	+19 45	5·53	G5	− 13·0	4	1·6	A	M.W. −11·5
143	30·4	− 1 4	5·93	F8	+ 4·3	4	0·7	S	
145	30·7	+12 40	6·40	F5	− 24·5	4	0·9	S	
149	31·1	+26 42	6·26	B8	+ 1·0	4	2·6	S	
205	40·7	+44 18	5·99	B8	− 6·6	4	1·4	S	
214	41·9	+19 1	6·06	A5	− 5	4	4·7	S	
249	48·0	+36 53	6·13	Ko	− 3·4	4	1·7	S	
254	49·3	+18 38	5·76	Ao	+ 8	4	6·0	S	
272	52·0	+45 18	6·24	Ko	+ 5·2	4	0·4	S	
275	52·7	+33 26	6·22	Ko	− 16·8	4	0·5	S	
277	53·0	+20 52	6·41	Ao	− 1	4	4·7	S	
289	57·3	+46 50	6·36	Fo	− 3·2	4	1·3	S	
HD 6226	58·2	+47 6	6·70	B8*p*	− 48·7	4	1·8	S	Ca^+ −18 km.
303	58·9	+29 9	6·08	F5	+ 4·1	4	1·3	S	
385	1 14·7	− 1 2	6·01	F5	+ 10·5	3	2·9	A	
407	20·1	+23 0	6·07	F5	...	4	...	A	Sp. Bin. Range 29 km. Plaskett, *J.R.A.S.C.*, 13, 193, 1919.
410	20·4	+34 4	6·26	F5	+ 15·1	4	1·6	A	
415	21·4	+33 50	6·28	F5	+ 14·5	4	1·0	A	
455	30·4	+14 9	6·20	B9	− 16	4	3·0	S	
457	30·5	+16 55	5·88	A5	− 2	4	3·0	S	
463	31·8	+11 38	5·63	Fo	− 5·5	5	1·7	S	
515	41·2	+16 55	6·46	Fo	− 8·1	4	1·9	S	
577	54·1	+20 34	6·06	Ko	− 1·8	4	1·3	S	
605	58·0	+25 28	5·68	F5	+ 14·4	4	0·9	S	
634	2 5·6	+31 3	6·20	Ao	...	9	...	A	Sp. Bin. Two spectra. Separation 135 km.
729	25·0	+19 25	6·14	Fo	+ 12·7	4	2·1	A	
738	26·8	+34 6	5·90	G5	− 2·7	3	0·7	A	
741	27·5	+14 36	6·07	F5	...	6	...	A	Sp. Bin. Range 27 km.
756	30·7	+38 18	5·94	F5	+ 0·4	4	1·9	S	
757	30·8	+31 10	6·16	Ko	+ 3·0	4	1·0	S	
760	31·1	+39 27	6·40	B8	...	3	...	S	Sp. Bin. Range 62 km.
761	31·1	+32 27	6·29	F5	+ 2·9	4	1·4	S	
770	32·7	+ 7 16	6·46	F5	+ 13·0	4	1·1	S	
816	40·1	+ 4 17	6·02	Fo	+ 19·9	4	1·7	S	
820	40·9	+35 9	6·34	F2	− 4·5	4	1·1	S	
856	47·6	+16 5	6·38	F2	+ 8·4	4	2·2	S	
927–8	59·6	+24 52	6·11	B8	+ 9·1	4	2·1	S	
958	3 7·1	+ 6 18	5·84	G5–A5	+ 3·6	4	1·9	A	
975	9·6	+32 29	6·34	Fo	+ 14·4	4	0·6	A	
995	14·0	+26 43	5·94	G5	− 1·1	4	2·2	A	
1039	21·8	+12 23	6·20	Ao	...	4	...	A	Sp. Bin. Range 32 km.
1067	25·4	+ 5 51	6·12	G5	+ 12·4	3	0·5	A	
1118	34·8	+25 0	6·15	Ao	...	4	...	S	Sp. Bin. Range 52 km. Plaskett, *J.R.A.S.C.*, 13, 193, 1919.

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates	P.E.	Obs.	
	h m	° ′							
1137	3 38·6	+20 37	6·03	B9	...	4	...	S	Sp. Bin. Range 54 km.
1194	45·7	+12 45	6·16	B9	+ 16·3	4	1·1	S	
1211	49·2	− 3 14	6·33	A3	+ 20·2	4	1·7	S	
1279	4 2·0	+14 54	5·94	Fo	+ 33·4	4	0·6	A	
1297	7·0	+22 9	6·16	B8	...	3	...	A	Sp. Bin. Range 24 km.
1319	10·1	+15 9	6·35	F5	+ 33·2	4	1·1	A	
1391	20·0	+15 43	6·39	F8	+ 33·7	4	1·1	A	M.W. +38·7.
1422	24·4	+15 25	5·70	Fo	+ 26·2	4	1·4	A	Sp. Bin. Victoria. Yerkes +20·7.
1432	26·2	+15 38	6·04	Fo	+ 26·7	4	2·3	A	
1459	30·5	+23 9	6·04	F2	+ 43·2	4	1·6	S	
1462	31·0	− 3 49	6·29	B9	+ 20	4	3·3	S	
1477	33·3	+25 2	6·27	A3	+ 17	4	2·5	S	
1499	37·2	+23 55	6·18	Go	+ 9·7	4	1·2	S	
1554	46·5	+27 44	5·91	F2	+ 37	4	5·0	S	
1566	49·0	+19 20	6·24	Fo	+ 31	4	2·8	S	
1591	52·2	− 1 14	6·23	F2	+ 11·6	4	2·1	S	
1691	5 6·6	+ 0 55	6·07	F5	− 19·4	5	3·3	A	
1822	23·4	+29 7	6·24	F5	+ 12·6	4	0·9	A	
1889	30·3	+25 53	6·32	F5	+ 2·6	4	2·1	S	
1908	31·5	+10 58	6·10	Ko	−118·8	4	0·9	S	
1967	38·0	− 6 51	5·98	F5	− 11	3	...	S	
1978	39·7	+ 3 58	6·14	Fo	+ 7·0	4	1·5	S	
2076	51·2	+11 30	6·08	G5	+ 21·5	4	1·6	S	
2111	54·7	+27 35	6·08	B8*p*	+ 16·6	4	1·6	S	
2122	56·3	+32 38	6·23	F5	+ 34·1	4	0·7	S	
2137	58·2	+37 58	6·40	F8	+ 5·1	4	1·3	S	
2139	59·0	+33 36	6·10	B9	+ 24·9	4	0·9	S	
2259	6 13·2	+ 9 6	6·38	Ko	− 14	4	3·6	A	Possibly Sp. Bin. Range 25 km.
2313	20·2	− 0 52	5·85	Fo	+ 44·1	4	0·5	A	
2339	22·7	+27 2	6·49	F5	− 6·9	3	1·3	A	
2452	34·9	+36 3	6·33	F5	+ 86·3	4	1·3	S	
2539	46·2	+38 59	6·06	F2	+ 2·7	4	1·2	S	
2542	46·4	+35 54	6·18	G5	+ 6·2	4	0·8	S	
2597	51·8	+12 2	6·16	Fo	+ 7·9	4	0·9	S	
2601	52·6	+26 13	6·10	F5	+ 6·4	4	0·8	S	
2779	7 14·3	+ 7 19	5·95	F8	+ 21·3	4	0·4	A	
2837	21·1	+20 27	5·88	Fo	− 0·2	4	2·6	A	
2918	31·2	+ 6 5	5·94	F8	+ 3·6	4	1·3	S	
2936	33·5	+32 14	6·14	Fo	+ 31·4	4	0·4	S	
3132	55·7	+25 22	6·20	Ao	− 8·7	4	1·0	S	
3134	55·8	+16 43	5·91	Ao	...	4	...	S	Sp. Bin. Range 49 km.
3184	8 3·1	+13 56	6·26	F5	− 8·2	4	1·0	A	
3222	7·4	+16 49	6·12	G5	− 20·9	5	1·9	A	
3589	55·9	+28 18	5·95	A5	...	4	...	S	Possibly Sp. Bin. Range 38 km.
3601	58·2	+28 18	6·34	Ao	− 26·1	3	4·9	S	
3625	9 2·7	+34 18	5·95	F8	+ 27·6	4	0·9	A	
3649	6·7	+ 5 53	6·21	Fo	+ 2·4	4	1·5	A	
3794	27·5	+ 2 19	6·15	F5	+ 27·6	4	2·0	A	
3879	41·3	+ 2 15	5·69	F2	+ 16·3	4	2·0	S	
3889	44·2	+21 39	6·01	Fo	+ 21	12	...	S	Sp. Bin. Two spectra. Separation > 80. Range 30 km. Plaskett, *D.A.O.*, 1, 163, 1920.
3893	45·3	+ 4 49	6·24	F5	+ 16·2	21	...	S	Sp. Bin. Two spectra. Separation 205 km.

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates	P.E.	Obs.	
	h m	° ′							
3936	9 52·8	+28 15	6·42	Fo	+ 35	4	...	S	Sp. Bin. Two spectra. Separation is difficult.
4006	10 8·2	+27 39	6·10	G5	...	4	...	A	Sp. Bin. Range 20 km.
4012	9·0	+21 41	6·12	F5	+ 17·3	3	1·8	A	
4079	18·1	+ 6 12	6·50	F2	+ 28·8	4	2·0	A	
4254	49·3	+26 1	6·18	Fo	+ 15·9	4	0·9	S	
4260	50·2	+25 17	6·30	A3	...	5	...	S	Possibly Sp. Bin. Range 52 km.
4410	11 20·7	+ 4 25	6·36	Fo	...	4	...	A	Sp. Bin. Range 34 km.
4433	25·3	+18 58	5·74	Ko	+ 30·5	3	2·0	A	
4455	29·3	+ 3 37	5·81	F5	− 0·4	4	1·4	A	M.W. +7·3
4533	43·9	+ 0 14	6·24	F8	+ 2·2	4	1·7	S	
4536	44·5	+35 29	5·76	F5	...	3	...	S	Sp. Bin. Range 65. Plaskett, *D.A.O.*, **1**, 163, 1920.
4642	12 6·9	+29 6	6·40	F2	− 9·9	4	1·3	A	
4680	12 13·5	+30 49	6·15	F5–A2	− 8	4	±3·6	S	Exceptionally poor spectrum. An impression of super-position of two spectra. M.W. −31 km.
4684	14·0	+26 34	6·39	A3	+ 10	4	3·9	S	
4705	17·2	+25 19	6·02	Ao	− 4·1	5	0·9	S	
4746	22·8	− 4 4	6·03	F2	...	4	...	A	Sp. Bin. Range 50 km. M.W. −14·0
4780	28·6	+24 50	6·14	A2	+ 2·6	4	1·8	S	
4793	30·2	+22 26	6·06	Ko	− 13·9	4	1·9	S	
4822	36·1	−12 28	5·98	F5	...	4	...	S	Sp. Bin. Range 35 km.
4821	36·1	−12 28	6·08	F2	...	4	...	S	Sp. Bin. Range 90 km.
5007	13 11·4	+19 36	6·54	G5	− 44·9	4	1·3	A	
5050	19·3	− 4 39	5·76	F2	...	5	...	A	Sp. Bin. Range 19 km.
5088	25·2	− 5 57	6·07	A5	...	4	...	A	Sp. Bin. Range 36 km.
5143	35·8	+31 31	6·08	G5	− 14·0	4	1·1	S	
5161	38·3	+35 30	5·98	Ko	− 26	4	2·7	S	
HD 120005	41·7	+31 22	6·55	F5	...	5	...	S	Sp. Bin. Range 45 km.
5180	42·0	+39 0	6·00	Ko	− 11·9	4	0·6	S	
5258	54·6	− 3 3	6·30	F5	− 8·6	3	1·5	S	
5270	57·6	+10 11	6·12	Go	− 21·1	4	1·9	S	
5342	14 11·3	− 2 44	6·03	Ao	+ 1·6	5	1·8	A	
5385	18·5	+ 8 54	6·64	F2	− 20	3	2·9	S	
5387	18·6	+25 48	6·15	F2	− 11·5	4	0·8	A	
5441	29·3	+37 24	6·44	F5	+ 0·9	4	0·6	A	
5476	36·0	+16 51	5·81	Ao	− 16	4	2·8	S	Yerkes from +2·2 to +30·0.
HD 129537	38·0	+15 8	6·61	F2	− 23·1	4	1·9	S	
5522	43·8	− 0 26	6·06	Ao	− 15	4	2·8	S	
5534	45·8	+24 20	5·81	Gop	+ 3·4	4	0·9	S	
5631	15 2·7	+ 5 52	6·22	G5	+ 4·5	4	1·7	A	
5639	4·1	+13 37	6·07	Ko	− 48·6	4	1·5	A	
5726	18·9	+39 56	5·85	K5	− 11·1	5	1·3	A	
5740	21·4	+19 59	6·29	Go	− 2·9	4	1·8	A	
5796	31·1	+11 35	6·11	G5	− 23·8	5	1·9	S	
5804	31·9	+16 27	5·88	Fo	− 6·0	4	1·6	S	
5813	32·8	+30 19	6·52	F5	− 12·8	4	2·2	S	
5827	34·9	+35 0	6·19	Ko	+ 4·8	4	0·6	S	
5853	39·1	+ 2 50	5·80	G5	+ 9·4	4	0·6	S	
5874	43·6	+14 6	6·10	Ko	− 55·2	4	1·0	S	
5875	43·7	− 3 31	5·61	A3	− 18·3	4	1·1	S	

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates	P.E.	Obs.	
	h m	° ′							
5913	15 49·1	+16 23	6·14	F2	− 3·0	4	1·8	S	
5919	49·8	+ 8 52	6·20	A2	− 2·1	4	3·7	S	
5983	59·6	+36 55	5·85	F5–A2	− 5	4	4·0	S	
5989	16 0·4	− 6 1	6·36	F5	− 10·6	7	0·6	A, S	According to A, possibly Sp. Bin. A plate not included in the mean gives −32 km.
6014	4·3	+ 6 41	6·02	G5	− 3·9	3	1·7	A	
6041	7·7	− 3 58	6·08	Ao	− 16·4	3	1·9	A	
6067	11·6	− 3 42	6·12	Fo	− 11·3	4	3·6	A	
6087	15·8	+21 23	6·14	G5	− 23·9	4	2·2	A	
6124	21·9	+ 2 35	6·18	G5	+ 1·0	4	0·9	A	
6144	25·1	− 7 17	6·39	A5	− 0·9	3	1·9	S	
HD 149504	30·2	+38 18	6·59	F5	− 49·4	4	1·0	S	
6181	33·2	+13 54	6·20	F2	− 21·2	4	1·1	S	
6239	43·5	+13 46	6·32	G5	+ 1·0	4	0·5	S	
6277	49·0	− 1 27	6·21	Fo	− 21	4	4·8	S	
6290	50·7	+13 47	6·16	F2	...	4	...	S	Sp. Bin. Range 32 km.
6333	58·2	+25 40	5·95	Ko	− 49·4	4	0·8	S	
6341	59·1	+13 44	5·86	Ao	− 37·8	4	0·3	S	M.W. −37.
6349	17 0·2	+ 0 49	5·94	F8	− 18·0	3	0·2	S	M.W. −18·6.
6352	0·4	+19 45	6·13	Ao	− 30·2	4	4·1	A	
6353	0·4	− 0 45	5·62	B3	...	4	...	A	Possibly Sp. Bin. Range 21 km. Ca^+ −26·2.
6367	3·1	− 0 57	6·02	Ao	− 17·7	5	4·2	A	
6469	18·4	+40 5	5·72	F8	...	4	...	A	Sp. Bin. Plaskett, *D.A.O.*, 1, 171, 1920.
6488	20·7	+38 41	6·42	F8	− 26·0	4	0·6	A	
6489	20·7	− 1 32	6·31	F5	− 23·8	3	4·5	A	
6499	22·0	+26 57	6·36	A5	− 28·7	4	2·1	A	
6524	26·4	+ 2 48	5·59	Gop	...	5	...	A	Sp. Bin. Harper, *D.A.O.*, 1, 297, 1921.
6541	29·1	+19 20	5·59	F5	− 58·6	4	0·6	A	M.W. −61·0.
6577	34·4	+13 23	6·29	F2	...	4	...	S	Sp. Bin. Range 37 km.
6609	39·5	+ 2 37	6·25	Ao	− 40·6	4	1·0	S	
6610	39·6	+ 2 37	6·64	Ao	− 25	4	3·0	S	
6619	41·9	+31 33	6·25	B9	+ 1·0	4	1·0	S	
6655	46·6	+22 20	5·91	A3	+ 4·0	4	1·3	S	
6676	49·6	+11 10	6·26	F2	...	4	...	S	Sp. Bin. Range 38 km.
6686	51·5	− 4 4	5·60	Ko	− 40·8	3	1·4	S	
6719	56·0	+ 6 16	6·18	B8	...	4	...	S	Probably Sp. Bin. Range 29 km.
6720	56·1	+19 31	6·42	B9	− 29	4	2·7	S	Ca^+ −23 km.
6722	56·4	+15 7	6·30	G5	+ 4·3	4	0·3	S	
6726	56·9	+33 14	6·07	K5	− 15·3	4	0·7	S	
6754	18 0·6	+23 56	6·25	Fo	− 37·8	3	1·3	A	
6764	1·5	+40 5	6·48	F5	− 1·0	4	1·2	A	
6776	3·2	+13 3	6·46	Ao	− 21·4	4	1·4	A	
6784	4·0	+14 16	6·30	A2	− 8·7	4	0·1	A	
6797	4·8	+ 3 6	5·73	F5	− 15·8	4	1·4	A	
6882	18·0	+23 14	5·66	K5	− 57·4	5	0·4	A	M.W. −57·4.
6898	19·8	− 1 38	6·11	F5	− 11·1	4	3·2	A	
6955	26·6	+16 52	5·67	Ao	− 8·2	4	1·7	A	
6957	26·8	− 1 4	5·81	A2	− 27·3	5	1·4	A	
7047	40·5	+23 29	6·17	F2	− 10·4	4	2·0	S	
7057	41·3	+37 30	5·87	A5	− 28	5	2·7	S	
7079	44·1	+23 24	6·04	F5	− 0·2	4	0·9	S	
7101	46·1	− 3 26	6·04	A3	+ 12·9	4	1·5	S	M.W. +12.

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates	P.E.	Obs.	
	h m	° ′							
7140	18 51·2	+33 51	6·08	G0	− 24·7	4	1·4	S	
7143	51·2	− 1 56	6·20	A0	− 22·6	4	2·5	S	
7163	53·5	+ 6 7	6·38	F5	− 9·4	4	1·9	S	
7171	54·5	+19 39	6·22	B8	− 1·0	3	3·0	S	
7204	57·3	+33 40	6·15	K0	− 25·8	4	0·3	S	
7209	57·6	− 3 51	5·55	A0	− 42·7	4	0·8	S	Yerkes −33·2.
7222	59·4	+21 7	6·50	F2	+ 4·0	3	1·8	S	
7250	19 2·5	+24 6	5·72	A5	− 20·5	4	1·2	A	
7260	3·5	+16 43	5·99	G5	...	29	...	A	Sp. Bin. Adams, Joy, Sanford, *P.A.S.P.*, **36**, 138, 1924. Range 27 km.
7308	12·0	+27 45	6·06	F8*p*	− 18·2	4	2·0	A	
7332	15·2	+11 21	6·02	A2	− 22·5	4	3·4	A	
7354	18·0	+ 9 43	6·25	F8	− 19·5	3	0·5	A	
7357	18·6	+14 44	6·56	A3	+ 4·0	4	1·8	A	
7389	21·8	+12 49	5·77	F5	− 34·6	4	1·6	A	M.W. −33·9.
7421	27·2	+26 24	5·96	K0	− 5·0	4	1·5	A	
7438	30·4	+ 2 42	6·50	F2	+ 3·4	4	1·8	S	
HD 184905	31·5	+43 43	6·58	A0*p*	− 11·7	4	0·7	S	
7452	31·8	+22 22	6·12	B9	− 31	4	3·0	S	
7454	31·9	−14 31	5·60	F8	− 25·3	4	1·5	S	
7456	32·1	+11 3	6·16	G5	− 0·1	4	0·6	S	
7460	32·5	− 4 52	5·52	F2	− 38·8	4	1·5	S	
HD 196885	35·1	+10 53	6·42	F8	− 28·3	4	1·2	S	
7493	37·5	+11 58	6·26	B9	− 31·8	4	1·8	S	
7501	38·8	+29 5	6·44	F0	− 26·2	4	0·8	S	
7505	39·3	+30 26	6·06	B9	− 30·6	4	2·4	S	
7540	43·6	+25 8	6·04	K0	− 16·7	4	0·6	S	
7542	43·7	+10 26	6·38	G0*p*	− 4·9	4	1·9	S	
7556	45·9	+28 12	6·29	B8	...	5	...	S	Probably Sp. Bin. Range 47 km.
7569	47·4	+11 23	6·18	G0	− 16·7	4	1·5	S	
7573	47·8	+24 44	5·67	F5–A2	+ 0·7	4	1·4	S	
7598	49·8	+ 6 53	5·97	A0	− 17·5	4	2·0	S	
7607	51·1	+29 56	6·36	B9	− 11	5	4·9	S	
7656	57·5	+24 31	5·75	B8	− 15·0	4	2·6	S	
7662	58·8	+18 14	6·14	K2	+ 9·5	5	±1·2	S	
7677	20 0·6	+22 55	6·41	A3	− 19·5	4	4·9	A	
7690	2·9	− 0 58	6·04	G5	− 4·6	4	2·3	A	
7693	3·1	+ 9 6	6·38	F5	− 32·3	4	1·1	A	
7719	7·8	+26 11	5·91	B8	...	4	...	A	Sp. Bin. M.W., *Ap. J.*, 35, 178, 1912.
7723	9·3	+23 56	6·48	A0	− 39·5	4	2·0	A	
7743	11·6	+33 26	5·78	G5	− 7·9	4	0·9	A	M.W. −10·6.
7769	14·8	+36 41	5·52	A0	...	4	...	A	Sp. Bin. Range 26 km. M.W., *Ap. J.*, **35**, 178, 1912.
7774	15·3	+13 14	5·96	A5	− 9·4	4	1·5	A	
7793	18·2	+14 13	6·22	F5	− 2·2	4	0·9	A	
7807	20·0	+37 10	5·68	B3	− 9	4	4·9	A	
7823	23·3	+34 0	6·41	F0	− 14·5	4	0·6	A	
7836	25·5	+10 34	5·92	A0	− 15·3	4	1·6	A	
7874	31·8	+25 34	6·29	A2	− 19·0	23	...	S	Sp. Bin. Range 116. Orbit, Pulkovo Obs. Circular No. 1, 1932.
7878	32·2	− 0 15	6·16	B8	− 23·3	4	1·9	S	
7885	33·4	+31 13	6·24	A0	− 15	5	4·2	S	

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates	P.E.	Obs.	
	h m	° ′							
7890	20 34·0	− 2 46	6·26	B9	...	5	...	S	Possibly Sp. Bin. Range 52 km.
7917	36·5	+29 27	6·09	Ao	...	4	...	S	Sp. Bin. Range 39 km.
7974	44·9	+ 7 30	6·23	Ao	− 29	28	...	S	Sp. Bin. Range 82 km. Period 1·1021.
8005	49·8	+33 3	5·68	K2	− 8·5	4	1·5	S	
8030	53·6	+10 28	5·61	Ko	− 2·1	4	0·8	S	
8051	57·3	+35 38	6·08	Ko	− 7·4	5	0·8	S	
8056	58·0	+ 1 8	6·50	F5	+ 7·3	4	1·0	S	
8059	58·7	− 6 13	5·89	F5	− 3·8	4	1·4	S	M.W. +0·6.
8095	21 5·0	+ 2 32	6·47	F2	− 45·0	4	1·7	A	
8116	8·8	+15 34	6·20	A5	− 27·0	4	4·5	A	
8157	15·4	+37 49	5·83	F2	− 15·2	4	0·9	A	
8189	19·3	+36 58	6·58	F8	− 27·2	4	2·4	A	
8250	30·1	+22 19	6·37	F8	+ 13·7	8	0·9	A, S	
8265	32·7	+ 6 10	6·05	Ao	− 8·0	4	2·5	S	
8274	34·2	+25 4	6·30	G5	− 13·5	4	0·5	S	
8292	37·6	+10 21	5·95	B8	+ 6·3	4	1·2	S	
8314	39·8	+14 19	6·10	Go	− 18·4	4	1·1	S	
8349	47·0	+39 5	6·19	B9	+ 0·3	4	1·5	S	
8354	48·0	+28 20	5·62	F5	+ 18·0	3	1·3	S	
8363	49·4	− 3 46	6·18	F8	− 15·5	4	2·1	S	
8364	49·6	+19 15	6·33	Ko	+ 4·5	4	0·9	S	
8376	53·0	− 5 54	6·21	F2	+ 0·6	4	1·2	S	
8401	58·0	− 7 1	5·60	G5	+ 30·2	4	1·8	S	
8441	22 3·7	+25 3	6·03	F8	...	4	...	A	Sp. Bin. Range 6 km. Young, *D.A.O.*, 1, 175, 1920.
8455	5·5	+19 8	6·07	Go	+ 40·2	4	1·4	A	
8459	5·8	+20 29	6·40	A2	+ 7·8	4	2·5	A	
8491	11·0	+ 8 3	6·03	Ao	...	4	...	A	Sp. Bin. Range 89 km.
8584	27·0	+29 2	6·32	A5	...	26	...	A	Sp. Bin. Range 167 km.
8586	27·7	+19 43	6·31	Fo	− 19·9	5	1·8	A	
8604	31·6	+35 4	6·20	Ko	− 15·4	5	0·8	S	
8653	38·7	+10 26	6·43	F5	− 2	4	3·4	S	
8681	44·5	+ 9 58	6·46	Fo	− 15	4	4·1	S	
8706	48·6	+39 37	6·24	B8	+ 6·5	4	1·3	S	
8715	49·9	+ 0 32	6·05	A3	+ 12·2	4	0·7	S	
8723	51·1	+35 50	5·63	B9	+ 0·9	4	0·6	S	
8735	53·3	− 1 57	6·40	F2	− 14·9	4	1·0	S	
8738	53·7	+ 6 48	6·28	Ao	− 2·4	4	2·0	S	
8776	59·0	+ 6 4	6·34	F2	+ 3·0	4	1·2	S	
8782	59·9	− 8 14	5·56	Fo	− 19	4	2·7	S	
8838	23 9·6	+29 13	6·42	F5	+ 9·6	4	2·0	A	
8840	10·4	− 4 3	5·55	A2	...	4	...	A	Sp. Bin. Range 57 km.
8845	11·0	+24 15	6·52	F2	+ 4·9	3	2·2	A	
8868	14·2	− 5 40	5·70	F2	...	11	...	A	Sp. Bin. Range 74 km. Adams, Joy, Sanford, *P.A.S.P.*, 36, 138, 1924.
8873	14·6	+34 15	6·14	B9	− 6·5	3	1·3	A	
8891	17·7	+20 16	6·22	Ao	...	4	...	A	Sp. Bin. Range 43 km. Adams, Joy, Sanford, *P.A.S.P.*, 36, 138, 1924.
8944	29·0	− 1 48	5·98	Ao	− 2·8	4	1·8	A	
8962	32·7	+43 52	5·86	B9	− 11	4	4·6	S	
8977	35·6	+36 9	6·30	F5	− 0·2	4	1·0	S	
9015	43·7	+ 1 39	6·42	F2	...	4	...	S	Sp. Bin. Range 32 km.

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates	P.E.	Obs.
	h m	° ′						
9024	23 44·6	+35 53	5·91	G5	+ 0·8	6	1·0	S
9048	50·0	+ 6 31	6·12	B9	+ 15·4	4	1·2	S
9086	56·6	+41 49	6·13	B9	− 15	4	3·4	S
9093	57·4	+ 7 56	5·78	Fo	+ 8·2	5	1·5	S

TABLE II

(*List* 2)

Star HR	α 1900	δ 1900	Vis. Mag.	Spec.	Rad. Vel. km./sec.	No. of Plates.	Probable Errors		Obs.	Lick Velocity km./sec.
							Mean	Plate		
	h m	° ′								
β Cas	0 3·8	+58 36	2·42	F5	+14·7	6	±0·9	±2·3	S	+11·8
ε And	33·3	+28 46	4·52	G5	−81·5	4	1·0	2·1	A	−83·4
β And	1 4·1	+35 5	2·37	M*a*	+ 0·6	7	1·3	3·5	S	+ 0·5
η Psc	26·1	+14 50	3·72	G5	+15·9	11	0·5	1·8	A, S	+15·3
α Ari	2 1·5	+22 59	2·23	K2	−12·6	13	0·6	2·1	A, S	−14·1
α Cet	57·1	+ 3 42	2·82	M*a*	−29·0	4	0·7	1·4	S	−25·3
α Tau	4 30·2	+16 18	1·06	K5	+56·5	18	0·4	1·6	A, S	+54·9
π^3 Ori	44·4	+ 6 47	3·31	F8	+22·4	8	0·5	1·6	A, S	+25·0
χ^1 Ori	5 48·5	+20 16	4·62	F8	−11·0	3	1·5	2·6	A	−14·1
α CMi	7 34·1	+ 5 29	0·48	F5	− 3·4	8	1·0	2·8	S	− 3·0
β Gem	39·2	+28 16	1·21	Ko	+ 7·2	8	0·4	1·7	A, S	+ 3·6
ε Leo	9 40·2	+24 14	3·12	G0*p*	+ 3·2	3	1·3	2·2	S	+ 4·5
η Leo	10 1·9	+17 15	3·58	A0*p*	− 0·7	3	0·4	0·7	A	+ 2·1
40 Leo	14·3	+19 59	4·97	F5	+11·2	4	0·9	1·8	A	+ 5·8
ι Leo	11 18·7	+11 5	4·03	F5	− 9·8	7	1·0	2·6	A, S	−10·0
β Vir	45·5	+ 2 20	3·80	F8	+ 3·7	10	0·4	1·5	A, S	+ 4·9
42 Com	13 5·1	+18 4	4·47	F5	−17·5	8	1·2	3·5	A, S	−17·5
β Com	7·2	+28 23	4·32	Go	+ 8·2	4	0·5	1·0	S	+ 5·4
τ Boo	42·5	+17 57	4·51	F5	−13·8	10	0·6	2·2	A, S	−15·7
α Boo	14 11·1	+19 42	0·24	Ko	− 2·3	33	0·4	2·6	A, S	− 5·4
σ Boo	30·3	+30 11	4·48	Fo	+ 1·0	10	0·7	2·7	A, S	+ 0·2
η CrB	15 19·1	+30 39	5·05	Go	− 3·9	4	1·9	3·9	A	− 6·6
ε Ser	45·8	+ 4 47	3·75	A2	− 9·1	10	0·8	2·6	S	− 9·8
γ Ser	51·8	+15 59	3·86	F5	+ 7·8	17	0·6	2·1	A, S	+ 6·9
δ Oph	16 9·1	− 3 26	3·03	M*a*	−19·8	10	0·9	3·8	A, S	−19·9
α Her	17 10·1	+14 30	3·48	M*b*	−34·0	16	0·5	2·0	A, S	−32·6
β Oph	38·5	+ 4 37	2·94	Ko	−13·3	3	1·5	2·6	S	−12·2
μ^1 Her	42·5	+27 47	3·48	G5	−14·4	4	0·9	1·7	S	−16·1
ν Her	54·7	+30 12	4·48	Fo	−21·9	12	0·8	3·0	A, S	−22·1
110 Her	18 41·4	+20 27	4·26	F5	+22·8	15	0·5	2·4	A, S	+22·2
α Sge	19 35·6	+17 47	4·37	Go	+ 1·5	3	1·1	1·9	S	+ 1·6
γ Aql	41·5	+10 22	2·80	K2	− 2·8	19	0·5	2·4	A, S	− 2·4
γ Cyg	20 18·6	+39 56	2·32	F8	− 4·8	6	0·9	2·2	S	− 5·4
41 Cyg	25·3	+30 2	4·09	F5*p*	−18·1	11	0·6	2·0	A, S	−18·6
δ Del	38·8	+14 43	4·53	A5	+ 6·3	11	0·5	1·6	A, S	+ 8·3
β Aqr	21 26·3	− 6 1	3·07	Go	+ 4·0	7	0·3	0·8	A, S	+ 6·8
μ^1 Cyg	39·6	+28 17	4·73	F5	+18·1	5	0·5	1·2	A	+18·5
ε Peg	39·3	+ 9 25	2·54	Ko	+ 3·1	16	0·5	2·0	A, S	+ 4·7
α Aqr	22 0·6	− 0 48	3·19	Go	+ 6·6	4	1·5	2·9	A	+ 7·6
ξ Peg	41·6	+11 40	4·31	F5	− 5·6	11	0·9	3·2	A, S	− 4·5
β Peg	58·9	+27 32	2·61	M*a*	+ 9·6	4	1·5	3·0	S	+ 8·7
ι Psc	23 34·8	+ 5 5	4·28	F8	+ 3·3	20	0·3	1·5	A, S	+ 5·6

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NOTE ON THE EFFECT OF NEGLECTING PREFERENTIAL MOTION IN DERIVING A MEAN PARALLAX FROM PECULIAR MOTIONS.

Alan Fletcher, M.A.

1. *Summary.*—The values of the mean cross proper motion $\bar{\tau}$ and mean residual radial velocity $\bar{V}$ which we insert in the formula $\bar{\pi} = 4{\cdot}74\bar{\tau}/\bar{V}$ are too high,* if preferential motion is present and has been neglected. In investigating the consequences it has been assumed that the velocity-ellipsoid is a prolate spheroid with its major axis parallel to the galactic plane. When the stars are uniformly distributed over the sky, $\bar{\tau}$ and $\bar{V}$ will usually be exaggerated in about the same ratio, so that $\bar{\pi}$ will not suffer greatly. The total effect on $\bar{\pi}$ depends on the angle between the directions of solar and preferential motion, and the approximate cancellation of the two contributions in practice is accidental. When, however, the stars show strong galactic concentration, the exaggeration of $\bar{V}$ is considerably increased, and that of $\bar{\tau}$ is usually somewhat diminished, so that the resulting value of $\bar{\pi}$ will almost always be too low. The corresponding absolute magnitude correction reaches about $+0^{m}{\cdot}5$ for stars showing fairly strong galactic concentration and preferential motion. The above results have a bearing on the problem of Cepheid luminosity.

2. *Radial Velocities.*—These present no difficulty. Let $\cosh p = c/a$, the ratio of the major and minor axes of the velocity-spheroid. Then $\bar{V}$ is exaggerated by the factor

$$f = (a^2 \sin^2 \chi + c^2 \cos^2 \chi)^{\frac{1}{2}}/a$$

$$= (1 + \sinh^2 p \cos^2 \chi)^{\frac{1}{2}} \qquad (1)$$

in a small region of the sky at a distance χ from the vertex V. The mean factor f_u in the case of uniform distribution over the sky has been shown by Eddington † to be

$$f_u = \tfrac{1}{2}(c/a + p/\sinh p).$$

The mean factor $f(0)$ in the case of maximum galactic concentration, assuming the stars to be uniformly distributed on the galactic circle, and the vertices to lie on that circle, is

$$f(0) = \frac{1}{2\pi}\int_0^{2\pi} f\, d\chi$$

* It is convenient, in comparing $\bar{\tau}/\bar{V}$ with what it would be if preferential motion were absent, to take the arbitrary diameter of the velocity-sphere of comparison equal to the minor axis of the velocity-spheroid.

† *Stellar Movements*, 1914, p. 157.

$$\left.\begin{aligned} &= 2\pi^{-1}Ec/a, \\ \text{where} \qquad \cos\theta &= a/c \end{aligned}\right\} \tag{2}$$

and E is the complete elliptic integral of the second kind with modulus sin θ.

Table I gives the mean percentage exaggeration $e = 100(f-1)$ in the two cases.

TABLE I

Radial Velocities

c/a	e_u	$e(0)$
1·0	0	0
1·2	7	10
1·4	14	21
1·6	22	32
1·8	30	43
2·0	38	54
2·2	46	66

Thus the values for completely uniform distribution are, within about two units, two-thirds of those for maximum galactic concentration. This fair constancy of $e_u/e(0)$ for preferential motion of normal intensity suggests that it may be sufficient for ordinary purposes to consider e as the product of two factors, one depending on c/a and the other on the degree of galactic concentration. In other words, approximations valid for small preferential motion may be useful.

With such an approximation, the equation (1) giving the regional factor of exaggeration may be written

$$f = 1 + \tfrac{1}{2}\sinh^2 p \cos^2\chi.$$

Now if (λ, β) are the galactic co-ordinates of the region, and $(\lambda', 0)$ those of the vertex, we have

$$\cos\chi = \cos\beta\cos(\lambda - \lambda'). \tag{3}$$

Thus

$$e = 100(f-1) = 50\sinh^2 p\cos^2\beta\cos^2(\lambda-\lambda').$$

But, if the stars are uniformly distributed in longitude, the mean value of $\cos^2(\lambda-\lambda')$ is $\frac{1}{2}$, and the mean value of e round a circle of galactic latitude β is therefore

$$e(\beta) = 25\sinh^2 p\cos^2\beta. \tag{4}$$

Thus the factor depending on galactic concentration is $\overline{\cos^2\beta}$, and the value 2/3 noticed above for $e_u/e(0)$ is merely the mean of $\cos^2\beta$ over the sky. Actually, (4) becomes quite inaccurate long before c/a reaches 2·2, where it would give $e(0) = 96$, instead of 66; the factor $\sinh^2 p$ is not adequate to represent the variation of e with c/a. But the factor $\cos^2\beta$ derived from (4) retains its usefulness longer; even for $c/a = 2{\cdot}2$, the approximate values of $e(\beta)$ given by $e(0)\cos^2\beta$ are never more than 6 units below the true values of $e(\beta)$ given in Table II (below).

This approximation, though useful, is not essential. The mean factor of exaggeration round a circle of galactic latitude β is, from (1) and (3),

$$f(\beta) = \frac{1}{2\pi}\int_0^{2\pi} \{1 + \sinh^2 p \cos^2 \beta \cos^2 (\lambda - \lambda')\}^{\frac{1}{2}} d\lambda$$

$$= 2\pi^{-1} E \sec \theta,$$

where

$$\tan \theta = \sinh p \cos \beta,$$

which reduces to (2) when $\beta = 0$. From this formula the following table, giving $e = 100(f - 1)$, has been computed.

TABLE II

Radial Velocities. Values of $e(\beta)$

c/a	$\beta = 0°$	15°	30°	45°	60°	75°	90°
1·0	0	0	0	0	0	0	0
1·2	10	10	8	5	3	1	0
1·4	21	20	16	11	6	2	0
1·6	32	30	25	17	9	3	0
1·8	43	41	34	24	13	4	0
2·0	54	51	43	31	17	5	0
2·2	66	62	52	38	21	6	0

3. *Cross Proper Motions.*—Consider a small region R of the sky at a distance χ from the vertex V, and let A be the solar apex. Then the velocity-ellipse for R has its major axis parallel to RV. Let ξ and η be the mean proper motions, freed from parallactic motion, along and perpendicular to RV, and let τ be the mean proper motion perpendicular to RA; in each case the mean without regard to sign is meant. Then we have

$$\tau^2 = \xi^2 \sin^2 ARV + \eta^2 \cos^2 ARV,$$

and the mean cross proper motion is exaggerated by the factor

$$f' = \tau/\eta = \{1 + (\xi^2/\eta^2 - 1) \sin^2 ARV\}^{\frac{1}{2}}.$$

But

$$\xi^2/\eta^2 - 1 = (c^2/a^2 - 1) \sin^2 \chi,*$$

and

$$\sin \chi \sin ARV = \sin \gamma \sin \psi,$$

where $\gamma = AV$ and $\psi = VAR$. Therefore

$$f' = (1 + \sinh^2 p \sin^2 \gamma \sin^2 \psi)^{\frac{1}{2}}. \qquad (5)$$

This formula holds whatever the distance-distribution of the stars, provided, of course, that there are in each shell of distance enough stars to yield a fair mean cross proper motion for the shell. The mean cross proper motions are exaggerated in the same ratio f' for the stars of each separate shell and for the whole set of stars in the region. Note that f' depends, for given p and γ, only on the apical longitude ψ of the region—*i.e.* on its longitude when the solar apex and antapex are taken as poles, and the meridians of

* Cf. *Stellar Movements*, p. 139, where, however, the sines and cosines should be interchanged.

0° and 180° longitude lie along the great circle through those poles and the vertices (also through the drift-apices of the two-drift theory).

The mean factor for the whole sky may be found by averaging f' over a large number of small regions only when the stars of each region have the same η and therefore the same mean parallax. This may reasonably be assumed when the stars are uniformly distributed over the sky, and also when the stars are uniformly distributed on the galactic circle. In other cases, which will not be considered here, it is necessary to make some assumption about the variation of mean parallax with galactic latitude.

In the first of the above cases the stars are uniformly distributed in apical longitude, and the mean factor of exaggeration is

$$f_u' = \frac{1}{2\pi}\int_0^{2\pi} f'\,d\psi$$

$$= 2\pi^{-1}E \sec\theta,$$

where

$$\tan\theta = \sinh p \sin\gamma$$

Columns 2–4 of Table III have been computed from this formula.

TABLE III

Cross Proper Motions. Values of $e_u' = 100(f_u' - 1)$

	Exact Formula			Approximate Formula		
c/a	$\gamma = 35°$	$\gamma = 45°$	$\gamma = 55°$	$\gamma = 35°$	$\gamma = 45°$	$\gamma = 55°$
1·0	0	0	0	0	0	0
1·2	4	5	7	4	5	7
1·4	7	11	15	8	11	15
1·6	12	17	22	12	18	23
1·8	16	24	31	17	25	32
2·0	21	31	39	22	32	42
2·2	26	38	48	28	40	51

Comparison with column 2 of Table I shows that e_u' counterbalances about four-fifths of e_u when $\gamma = 45°$, a fairly typical value.* The formula $\bar{\pi} = 4{\cdot}74\bar{\tau}/\bar{V}$ would give mean parallaxes falsified by the factor f_u'/f_u, and therefore usually slightly too small, for groups of stars showing negligible galactic concentration. The approximate cancellation is accidental, being due to the fact that the vertex usually lies in middle apical latitudes. The tables show that the cancellation is fairly complete when $\gamma = 54°$; the critical value for small preferential motion is, in fact, $\gamma = 54\frac{3}{4}°$, given by $\sin^2\gamma = 2/3$. For values of γ greater than about 54° the formula would give mean parallaxes very slightly too large.

The approximation, $f_u' = 1 + \frac{1}{4}\sinh^2 p \sin^2\gamma$, obtained by starting the expansion of f' in (5) and averaging over the sky, is poor. But that obtained by substituting the mean value $\frac{1}{2}$ for $\sin^2\psi$ in (5), namely,

$$f_u' = (1 + \tfrac{1}{2}\sinh^2 p \sin^2\gamma)^{\frac{1}{2}},$$

* Taking V to mean the vertex nearer to A.

is quite good, as may be seen from the computed values in columns 5–7 of Table III. A similar remark holds, but has not much point, for the radial velocities. For the cross proper motions, however, the approximation is valuable, as will be seen in the next paragraph.

Consider now the case when the stars are distributed uniformly on the galactic circle. If VA makes an angle α with the galactic circle at V, and R lies on the galactic circle, we have

$$\cos\alpha\cos\gamma = \sin\gamma\cot\chi - \sin\alpha\cot\psi. \tag{6}$$

The mean value of f' round the galactic circle is

$$f'(0) = \frac{1}{2\pi}\int_0^{2\pi}(1+\sinh^2 p\sin^2\gamma\sin^2\psi)^{\frac{1}{2}}d\chi.$$

This elliptic integral can be reduced in the usual way ; but the approximation mentioned in the preceding paragraph saves us the necessity of performing the reduction, unless great accuracy is required. For the mean value of $\sin^2\psi$ round the galactic circle is, with normal values of α and γ, less than $\frac{1}{2}$, and the approximation is better than before. It is not difficult to show that the integral giving the mean value of $\sin^2\psi$, namely,

$$\overline{\sin^2\psi} = \frac{1}{2\pi}\int_0^{2\pi}\sin^2\psi\,d\chi, \quad \text{with the condition (6),}$$

reduces to

$$\overline{\sin^2\psi} = \frac{\sin\alpha(\sin\alpha+\sin\gamma)}{(1+\sin\alpha\sin\gamma)^2}, \tag{7}$$

which may also be written

$$\overline{\sin^2\psi} = \frac{\{1+\cot^2\beta_0\sin^2(\lambda_0-\lambda')\}^{-1}+\sin|\beta_0|}{(1+\sin|\beta_0|)^2},$$

where (λ_0, β_0) are the galactic co-ordinates of the apex, and $(\lambda', 0)$, as before, those of the vertex. Taking $\alpha = 35°$, $\gamma = 45°$, values sufficiently typical for the purpose of illustration, we find from (7) that the mean value of $\sin^2\psi$ is 0·372 ; by inserting this in (5) we find the approximate values given in column 5 of Table IV, where, in addition, some other values obtained above have been brought together, and the ratio $f'(0)/f(0)$ has been converted into magnitude-correction $\Delta M(0)$. The higher values of $e'(0)$ probably require reduction by 1 or 2 units, in order to be correct to the order given.

A glance at the table will show how neglect of preferential motion, while fairly harmless in the case of a group of stars uniformly distributed over the sky, may have comparatively important consequences in the case of a group showing strong galactic concentration. In the latter case a correction $\Delta M(0)$* should be applied, in order to compensate for the neglect of preferential motion, to absolute magnitudes derived through the formula $\bar{\pi} = 4{\cdot}74\bar{\tau}/\bar{V}$; it is supplementary to, and independent of, the correction depending on the dispersion of the stars in distance, but, of course, requires modification if

* $\Delta M(0)$ depends on α and γ ; the values in Table IV are merely typical.

the velocity-ellipsoid has three unequal axes.* The results obtained do not necessarily apply when the formula, $\varpi = 4{\cdot}74\tau/V$, where ϖ, τ and V are geometric means, has been used.

TABLE IV

c/a	R.V. e_u	C.P.M. e_u' ($\gamma = 45°$)	R.V. $e(0)$	C.P.M. $e'(0)$ ($\alpha = 35°, \gamma = 45°$)	$\Delta M(0)$
					m
1·0	0	0	0	0	+0·0
1·2	7	5	10	4	+0·1
1·4	14	11	21	9	+0·2
1·6	22	17	32	14	+0·3
1·8	30	24	43	19	+0·4
2·0	38	31	54	25	+0·5
2·2	46	38	66	31	+0·5

4. *Two-Drift Theory.*—The analysis on the two-drift theory is not entirely intractable. It is not difficult, for instance, to show that on this theory also the regional exaggeration of $\bar{V}$ depends only on χ, and that of $\bar{\tau}$ only on ψ. But the expression for the mean exaggeration of $\bar{\tau}$ comes out in terms of confluent hypergeometric functions in the case of a distribution uniform over the sky, and the expressions in the case of maximum galactic concentration have not been examined at all. In any case, the ellipsoidal theory is the obvious one to use for the purpose of obtaining rough estimates of the effect of neglecting preferential motion.

5. *Cepheids.*—It is hoped that the above calculations may be of some general value ; they were, however, originally made in order to find out to what extent the determination of Cepheid parallaxes from peculiar motions might be vitiated by neglect of preferential motion. The galactic concentration of the Cepheids † is very strong, and counts as little short of maximum concentration for the present purpose. How strongly the Cepheids show preferential motion the proper motions are scarcely adequate to decide, and the radial velocity material is still none too abundant ; all that can be said is that preferential motion seems to exist in fair strength. After allowing for a reasonable amount of solar motion and galactic rotation, the radial velocities collected by Lundmark ‡ show considerably greater scatter around longitudes $\lambda = 150°$ and 330° than around $\lambda = 60°$ (around $\lambda = 240°$ the list contains few stars). While any numerical conclusion is very likely to be affected by fresh accession of observational evidence, it seems probable, at the moment, that c/a may be about 1·6 or 1·8. This would imply, for normal apex and vertex, a supplementary correction of $+0^m{\cdot}3$ or $+0^m{\cdot}4$ to absolute magnitude ; for actual apex and vertex, possibly one slightly smaller, though still positive. It may be noticed that the correction to Shapley's earlier zero-point for the photographic period-luminosity curve, suggested

* Or is a prolate spheroid with its major axis not sufficiently nearly parallel to the galactic plane.

† This paragraph refers only to the Cepheids with periods greater than a day.

‡ *Lund Medd. II*, 60, p. 16, 1931.

by the recent unfinished discussion between Gerasimovič and Nordström,* comes to about $+0^{m}.6$ or $+0^{m}.7$ from parallactic motion, and $+1^{m}.4$ or somewhat less from peculiar motions. Consequently, as matters stand at the moment, the application of such a correction as $\Delta M(0)$ would, if anything, widen the gap already existing between the two determinations, by increasing still further the large zero-point correction derived (with considerable uncertainty) from peculiar motions. It should be remembered, however, that the velocity-ellipsoid for Cepheids may possibly have three appreciably unequal axes.

THE VARIATION OF ECCENTRICITY AND SEMI-AXIS MAJOR FOR THE ORBIT OF A SPECTROSCOPIC BINARY.

M. H. H. Walters, B.A.

1. *Introduction.*—This paper is an investigation of the effect of stellar encounters, over a long period of time, on the eccentricity and length of axis of the orbit of a spectroscopic binary star (for which initially both these quantities are relatively small). It will be convenient first to consider the effect produced by one encounter, assuming the (hyperbolic) orbit of the passing star to approximate to a straight line—say with eccentricity greater than 5 ; we shall also assume that its closest approach to the binary system is large compared with the axis of the binary orbit. We shall then proceed to evaluate the average amounts of the effects on eccentricity and axis, assuming encounters to take place with equal frequency in all directions and all planes for any given distance of closest approach and velocity ; then we shall integrate for all values of the distance and velocity between appropriate limits, having regard to the number of encounters varying with these two quantities. This will lead to our final results of the total effect produced during the whole history of a spectroscopic binary.

2. *General Equations of Motion.*—Denote the stars of the binary system by m_1 and m_2, and let $m_1=\beta m_2$, $(\beta \leqslant 1)$, $M=m_1+m_2$; denote the disturbing star by m. Take rectangular axes, $OXYZ$, and let the co-ordinates of m_1 be $(X_1Y_1Z_1)$, etc. Let G be the centre of gravity of the binary, and suppose $Gm_1=r_1$, $Gm_2=r_2$, $Gm=R$, $mm_1=R_1$, $mm_2=R_2$ and $m_1m_2=r$.

The equations of motion for the three bodies are

$$\left.\begin{aligned} m_1X_1'' &= k^2m_1m_2\frac{X_2-X_1}{r^3}+k^2m_1m\frac{X-X_1}{R_1^3}\\ m_2X_2'' &= k^2m_1m_2\frac{X_1-X_2}{r^3}+k^2mm_2\frac{X-X_2}{R_2^3}\\ mX'' &= k^2mm_1\frac{X_1-X}{R_1^3}+k^2mm_2\frac{X_2-X}{R_2^3} \end{aligned}\right\}. \qquad (1)$$

* Gerasimovič, *A.J.*, 951, 1931, and *Lund Circ.*, 4, 1931 ; Nordström, *Lund Circ.*, 2 and 4, 1931.

Transferring to parallel axes through G, $(X_0 Y_0 Z_0)$, we have

$$MX_0 = m_1 X_1 + m_2 X_2 ;$$

put

$$x_1 = X_1 - X_0, \qquad x_2 = X_2 - X_0, \qquad x = X - X_0,$$

so that

$$m_2 X_2 = m_2 X_0 - m_1 x_1, \quad \text{or} \quad m_2 x_2 = -m_1 x_1, \qquad x_2 = -\beta x_1.$$

We write now, from (1), adding the first two equations,

$$X_0'' = \frac{k^2 m_1 m}{M}\frac{x - x_1}{R_1^3} + \frac{k^2 m_2 m}{M}\frac{x - x_2}{R_2^3}. \tag{2}$$

We have also, from (1) and (2),

$$m_1 x_1'' = m_1 X_1'' - m_1 X_0''$$
$$= \frac{k^2 m_1 m_2 (x_2 - x_1)}{r^3} + \frac{k^2 m m_1 (x - x_1)}{R_1^3} - \frac{k^2 m_1^2 m (x - x_1)}{M R_1^3} - \frac{k^2 m m_1 m_2 (x - x_2)}{M R_2^3}.$$

Writing $x_2 = -\beta x_1$, $m_2 r = M r_1$, and remembering that $(1+\beta)m_2 = M$, we have

$$m_1 x_1'' = -\frac{k^2 m_1 m_2^3}{M^2}\frac{x_1}{r_1^3} + \frac{k^2 m m_1 m_2}{M}\left[\frac{x - x_1}{R_1^3} - \frac{x + \beta x_1}{R_2^3}\right].$$

Since $r_1^2 = x_1^2 + y_1^2 + z_1^2$, etc.,

$$-\frac{x_1}{r_1^3} = \frac{\partial}{\partial x_1}\left(\frac{1}{r_1}\right), \qquad -\frac{(x_1 - x)}{R_1^3} = \frac{\partial}{\partial x_1}\left(\frac{1}{R_1}\right) \quad \text{and} \quad -\frac{(x + \beta x_1)}{R_2^3} = \frac{1}{\beta}\frac{\partial}{\partial x_1}\left(\frac{1}{R_2}\right),$$

so we can write

$$m_1 x_1'' = \frac{k^2 m_1 m_2^3}{M^2}\frac{\partial}{\partial x_1}\left(\frac{1}{r_1}\right) + \frac{k^2 m m_2^2}{M}\frac{\partial}{\partial x_1}\left(\frac{\beta}{R_1} + \frac{1}{R_2}\right). \tag{3}$$

Again, from (1) and (2), we have

$$m x'' = \frac{k^2 m_1 m (x_1 - x)}{R_1^3} + k^2 m_2 m \frac{x_2 - x}{R_2^3} - \frac{k^2 m_1 m^2 (x - x_1)}{M R_1^3} - \frac{k^2 m^2 m_2 (x - x_2)}{M R_2^3},$$

or

$$x'' = \frac{k^2 m m_2 (M + m)}{M}\frac{\partial}{\partial x}\left(\frac{\beta}{R_1} + \frac{1}{R_2}\right). \tag{4}$$

Now write $F \equiv \dfrac{\beta}{R_1} + \dfrac{1}{R_2}$, $S \equiv$ cosine of the angle between Gm_1 and Gm, so that $R_1^2 = R^2 + r_1^2 - 2 r_1 R S$. We have

$$\left.\begin{aligned} \frac{1}{R_1} &= \frac{1}{R}\left[1 - \frac{2r_1}{R}S + \frac{r_1^2}{R^2}\right]^{-\frac{1}{2}} \\ &= \frac{1}{R}\left[1 + \frac{r_1}{R}P_1(S) + \frac{r_1^2}{R^2}P_2(S) + \ldots\right]. \end{aligned}\right\} \tag{5}$$

Similarly

$$\frac{1}{R_2} = \frac{1}{R}\left[1 - \frac{r_2}{R}P_1(S) + \frac{r_2^2}{R^2}P_2(S) - \ldots\right].$$

We have $m_2 r_2 = m_1 r_1$, so that $r_2 = \beta r_1$, and

$$RF = \beta + \frac{\beta r_1}{R} P_1 + \frac{\beta r_2^2}{R^2} P_2 + \ldots + \frac{\beta r_1^j}{R^j} P_j + \cdots$$

$$+ 1 - \frac{\beta r_1}{R} P_1 + \frac{\beta^2 r_1^2}{R^2} P_2 + \ldots + \frac{(-\beta)^j r_1^j}{R^j} P_j + \cdots$$

$$F = \frac{1+\beta}{R} + \beta(1+\beta)\left[\frac{r_1^2}{R^3} P_2 + (1-\beta)\frac{r_1^3}{R^4} P_3 + (1-\beta+\beta^2)\frac{r_1^4}{R^5} P_4 + \ldots\right]. \quad (6)$$

Define

$$\Omega \equiv \frac{r_1^2}{R^3} P_2 + (1-\beta)\frac{r_1^3}{R^4} P_3 + \cdots \quad (7)$$

so that

$$F = (1+\beta)\left(\frac{1}{R} + \beta\Omega\right)$$

and

$$x_1'' = \frac{k^2 m_2^3}{M^2} \frac{\partial}{\partial x_1}\left(\frac{1}{r_1}\right) + \frac{k^2 m}{\beta(1+\beta)} \beta(1+\beta)\frac{\partial \Omega}{\partial x_1},$$

$$x'' = \frac{k^2(M+m)}{1+\beta} \frac{\partial}{\partial x}\left(\frac{1+\beta}{R}\right) + \frac{k^2(M+m)}{1+\beta} \beta(1+\beta)\frac{\partial \Omega}{\partial x}.$$

If we put $k^2 m_2^3 / M^2 = \mu_1$ and $k^2(M+m) = \mu$, we can write

$$x_1'' = \mu_1 \frac{\partial}{\partial x_1}\left(\frac{1}{r_1}\right) + k^2 m \frac{\partial \Omega}{\partial x_1}, \quad (8)$$

$$x'' = \mu \frac{\partial}{\partial x}\left(\frac{1}{R}\right) + k^2 \beta (M+m)\frac{\partial \Omega}{\partial x}, \quad (9)$$

where

$$\Omega = \frac{m_2}{m_1 M}\left(\frac{m_1}{R_1} + \frac{m_2}{R_2} - \frac{M}{R}\right).$$

The first terms of (8) and (9) give respectively the undisturbed elliptic orbit of m_1 about G and the motion of m about a mass M at G. We have a disturbing function $k^2 m\Omega$ for the motion of m_1, and a disturbing function $k^2\beta(M+m)\Omega$ for the motion of m.

3. *Elliptic and Hyperbolic Co-ordinates.*—Let the elements of the orbit of m_1 be $a_1 e_1 \omega_1 \chi_1 \phi_1 i_1$ and those of the orbit of m, $ae\omega\chi\phi i$. We note that the separation of the binary will be Ma_1/m_2. Let A_1 and A be the apses of the orbits and $A_1\hat{G}m_1 = v_1$, $A\hat{G}m = v$.

The equation for an ellipse is

$$r_1 = a_1(1 - e_1 \cos u_1)$$

and that for a hyperbola is

$$R = a(e \cosh D - 1) \quad (10)$$

with the usual notation, and

$$u_1 - e_1 \sin u_1 = n_1 t + \chi_1$$

$$e \sinh D - D = nt + \chi \quad (11)$$

Also we have the relations

$$\left.\begin{aligned}\cos v_1 &= \frac{\cos u_1 - e_1}{1 - e_1 \cos u_1}, \qquad \sin v_1 = \frac{(1-e_1^2)^{\frac{1}{2}} \sin u_1}{1 - e_1 \cos u_1}, \qquad \cos v = \frac{e - \cosh D}{e \cosh D - 1} \\ \text{and} \qquad \sin v &= \frac{(e^2-1)^{\frac{1}{2}} \sinh D}{e \cosh D - 1}\end{aligned}\right\}. \tag{12}$$

Now $GA = a(e-1) = p$, say, the closest approach of m to G; so that

$$R = \frac{p(e+1)}{1 + e \cos v}. \tag{13}$$

Differentiate equation (7) with respect to r_1, R and S, multiply by R^2, R^2 and R^2/r_1 respectively, and substitute in the right-hand sides from (10) and (13) for r_1 and R; then

$$\left.\begin{aligned}R^2\frac{\partial \Omega}{\partial r_1} &= \frac{2a_1}{p}(1 - e_1 \cos u_1)\left(\frac{e \cos v + 1}{e+1}\right)P_2 \\ &\qquad + 3(1-\beta)\left(\frac{a_1}{p}\right)^2(1 - e_1 \cos u_1)^2\left(\frac{e \cos v + 1}{e+1}\right)^2 P_3 + \cdots \\ R^2\frac{\partial \Omega}{\partial R} &= -3\left(\frac{a_1}{p}\right)^2(1 - e_1 \cos u_1)^2\left(\frac{e \cos v + 1}{e+1}\right)^2 P_2 \\ &\qquad - 4(1-\beta)\left(\frac{a_1}{p}\right)^3(1 - e_1 \cos u_1)^3\left(\frac{e \cos v + 1}{e+1}\right)^3 P_3 - \cdots \\ \frac{R^2}{r_1}\frac{\partial \Omega}{\partial S} &= \frac{a_1}{p}(1 - e_1 \cos u_1)\left(\frac{e \cos v + 1}{e+1}\right)P_2' \\ &\qquad + (1-\beta)\left(\frac{a_1}{p}\right)^2(1 - e_1 \cos u_1)^2\left(\frac{e \cos v + 1}{e+1}\right)^2 P_3' + \cdots\end{aligned}\right\}. \tag{14}$$

4. *Expression of* Ω *in Terms of the Elements.*—We proceed to find an expression for S in terms of the angular elements of the two orbits, together with v and v_1. The inclinations of the orbits to the xy plane are i and i_1; let great circles on a unit sphere in the planes of these orbits be N_1Vm_1, NVm, where N_1 and N are the ascending nodes on the xy plane. We have $N_1N = \phi - \phi_1$, where ϕ and ϕ_1 are the longitudes of the nodes, $N_1m_1 = v_1 + \omega_1$, $Nm = v + \omega$. Suppose $N_1V = \epsilon_1$, $NV = \epsilon$, $N_1\hat{V}N = \psi$, $mm_1 = \theta$.

We have

$$\begin{aligned}S &= \cos(\omega + v - \epsilon)\cos(\omega_1 + v_1 - \epsilon_1) + \cos\psi \sin(\omega + v - \epsilon)\sin(\omega_1 + v_1 - \epsilon_1) \\ &= \cos(\omega+v)\cos(\omega_1+v_1)[\cos\epsilon\cos\epsilon_1 + \cos\psi\sin\epsilon\sin\epsilon_1] \\ &\quad + \cos(\omega+v)\sin(\omega_1+v_1)[\cos\epsilon\sin\epsilon_1 - \cos\psi\sin\epsilon\cos\epsilon_1] \\ &\quad + \sin(\omega+v)\cos(\omega_1+v_1)[\sin\epsilon\cos\epsilon_1 - \cos\psi\cos\epsilon\sin\epsilon_1] \\ &\quad + \sin(\omega+v)\sin(\omega_1+v_1)[\sin\epsilon\sin\epsilon_1 + \cos\psi\cos\epsilon\cos\epsilon_1].\end{aligned} \tag{15}$$

Now we have at once

$$\cos\epsilon\cos\epsilon_1 + \cos\psi\sin\epsilon\sin\epsilon_1 = \cos(\phi - \phi_1). \tag{16}$$

Produce NV, N_1V to K and K_1 respectively, such that $NK = N_1K_1 = 90°$, and let the great circle KK_1 cut NN_1 in W.

Then from ΔKVK_1

$$\sin\epsilon \sin\epsilon_1 + \cos\psi \cos\epsilon \cos\epsilon_1 = \cos KK_1,$$

from ΔKK_1N_1

$$= \cos KN_1 \cos K_1N_1 + \sin KN_1 \sin K_1N_1 \cos KN_1K_1,$$

from ΔKN_1N

$$= \frac{\sin i \cos KN_1K_1}{\sin KN_1N}.$$

From ΔKN_1N we have

$$\sin(\phi - \phi_1)\cot 90° - \sin i \cot KN_1N = -\cos i \cos(\phi - \phi_1)$$

or

$$\cot KN_1N = \cot i \cos(\phi - \phi_1),$$

so that

$$\frac{\cos KN_1K_1 \cos i_1 - \sin KN_1K_1 \sin i_1}{\cos KN_1K_1 \sin i_1 + \sin KN_1K_1 \cos i_1} = \cot i \cos(\phi - \phi_1)$$

and

$$\frac{\cos KN_1K_1}{\sin KN_1N} = \cot i \cos(\phi - \phi_1)\cos i_1 + \sin i_1.$$

Finally,

$$\sin\epsilon \sin\epsilon_1 + \cos\psi \cos\epsilon \cos\epsilon_1 = \cos i \cos i_1 \cos(\phi - \phi_1) + \sin i \sin i_1. \quad (17)$$

From ΔK_1VN we have

$$\cos\epsilon \sin\epsilon_1 - \sin\epsilon \cos\epsilon_1 \cos\psi = \cos K_1N = \sin(\phi - \phi_1)\cos i_1. \quad (18)$$

From ΔKVN_1 we have

$$\sin\epsilon \cos\epsilon_1 - \cos\epsilon \sin\epsilon_1 \cos\psi = \cos KN_1 = -\sin(\phi - \phi_1)\cos i. \quad (19)$$

Substituting from (16), (17), (18) and (19) in (15) we have

$$\begin{aligned} S = {} & \cos(\omega + v)\cos(\omega_1 + v_1)\cos(\phi - \phi_1) + \cos(\omega + v)\sin(\omega_1 + v_1)\sin(\phi - \phi_1)\cos i_1 \\ & - \sin(\omega + v)\cos(\omega_1 + v_1)\sin(\phi - \phi_1)\cos i \\ & + \sin(\omega + v)\sin(\omega_1 + v_1)[\cos i \cos i_1 \cos(\phi - \phi_1) + \sin i \sin i_1]. \end{aligned} \quad (20)$$

With the aid of (10), (11), (12) and (20) we now have Ω expressible as a function of the twelve elements and t, so that we may proceed, by the method of variation of constants, to examine the variations in a_1 and e_1 during the passage of m.

5. *Equations of Variation of Constants.*—First let us consider e_1; we have, with $k^2m\Omega$ as our disturbing function, the well-known equation,

$$\frac{de_1}{dt} = \frac{1 - e_1^2}{n_1a_1^2e_1}k^2m\frac{\partial\Omega}{\partial\chi_1} - \frac{(1 - e_1^2)^{\frac{1}{2}}}{n_1a_1^2e_1}k^2m\frac{\partial\Omega}{\partial\omega_1}. \quad (21)$$

Regarding $\Omega \equiv \Omega(r_1RS)$, we have

$$\frac{\partial\Omega}{\partial\chi_1} = \left[\frac{\partial\Omega}{\partial r_1}a_1e_1 \sin u_1 + \frac{\partial\Omega}{\partial S}\frac{\partial S}{\partial v_1}\frac{\partial v_1}{\partial u_1}\right]\frac{\partial u_1}{\partial\chi_1}. \quad (22)$$

Now from (11) and (12) we see that

$$\frac{\partial u_1}{\partial \chi_1} = \frac{1}{1 - e_1 \cos u_1}$$

and

$$-\sin v_1 \frac{\partial v_1}{\partial u_1} = \frac{(1 - e_1 \cos u_1) + (\cos u_1 - e_1)e_1}{(1 - e_1 \cos u_1)^2}(-\sin u_1),$$

so that

$$\left.\begin{aligned} \frac{\partial v_1}{\partial u_1} &= \frac{(1 - e_1^2)^{\frac{1}{2}}}{1 - e_1 \cos u_1} \\ \text{and} \qquad \frac{\partial \Omega}{\partial \omega_1} &= \frac{\partial \Omega}{\partial S}\frac{\partial S}{\partial v_1} \end{aligned}\right\}. \tag{23}$$

Substituting from (22) and (23) in (21) we have

$$\begin{aligned} \frac{de_1}{dt} &= \frac{k^2 m}{n_1 a_1^2 e_1}\left\{\frac{(1 - e_1^2)}{1 - e_1 \cos u_1} a_1 e_1 \sin u_1 \frac{\partial \Omega}{\partial r_1} + \frac{\partial \Omega}{\partial S}\frac{\partial S}{\partial v_1}(1 - e_1^2)^{\frac{1}{2}}\left[\frac{1 - e_1^2}{(1 - e_1 \cos u_1)^2} - 1\right]\right\} \\ &= \frac{k^2 m}{n_1 a_1}\frac{(1 - e_1^2)^{\frac{1}{2}}}{1 - e_1 \cos u_1}\left\{(1 - e_1^2)^{\frac{1}{2}} \sin u_1 \frac{\partial \Omega}{\partial r_1} + \frac{2 \cos u_1 - e_1(1 + \cos^2 u_1)}{r_1}\frac{\partial \Omega}{\partial S}\frac{\partial S}{\partial v_1}\right\}. \end{aligned} \tag{24}$$

Substituting from (14) we obtain

$$\begin{aligned} \frac{n_1 a_1 R^2}{k^2 m}\frac{de_1}{dt} = \frac{(1 - e_1^2)^{\frac{1}{2}}}{1 - e_1 \cos u_1}\Bigg\{&\Bigg[2(1 - e_1^2)^{\frac{1}{2}} \sin u_1 P_2 \\ &+ \{2 \cos u_1 - e_1(1 + \cos^2 u_1)\}\frac{\partial S}{\partial v_1}P_2'\Bigg] \times \frac{a_1}{p}(1 - e_1 \cos u_1)\frac{e \cos v + 1}{e + 1} \\ &+ \Bigg[3(1 - e_1^2)^{\frac{1}{2}} \sin u_1 P_3 + \{2 \cos u_1 - e_1(1 + \cos^2 u_1)\}\frac{\partial S}{\partial v_1}P_3'\Bigg] \\ &\times \left(\frac{a_1}{p}\right)^2 (1 - \beta)(1 - e_1 \cos u_1)^2\left(\frac{e \cos v + 1}{e + 1}\right)^2 + \cdots\Bigg\}. \end{aligned} \tag{25}$$

If we now write $(e \cos v + 1)/(e + 1) = (e - 1)/(e \cosh D - 1)$ and remember that e_1, a_1/p and $1/e$ are small quantities of the first order, denoting a_1/p by α_1 and $1/e$ by f, we have

$$\frac{e - 1}{e \cosh D - 1} = \frac{1}{\cosh D}\left[1 - \frac{f(\cosh D - 1)}{\cosh D} - \frac{f^2(\cosh D - 1)}{\cosh^2 D} + \cdots\right],$$

and hence

$$\begin{aligned} \frac{n_1 a_1 p^2}{k^2 m}\frac{de_1}{dt} = 2&\left(\sin u_1 P_2 + \cos u_1 \frac{\partial S}{\partial v_1}P_2'\right)\frac{\alpha_1}{\cosh^3 D} - (1 + \cos^2 u_1)\frac{\partial S}{\partial v_1}P_2'\frac{e_1 \alpha_1}{\cosh^3 D} \\ &+ (1 - \beta)\left(3 \sin u_1 P_3 + 2 \cos u_1 \frac{\partial S}{\partial v_1}P_3'\right)\frac{\alpha_1^2}{\cosh^4 D} \\ &- 6\left(\sin u_1 P_2 + \cos u_1 \frac{\partial S}{\partial v_1}P_2'\right)\frac{\alpha_1 f(\cosh D - 1)}{\cosh^4 D} + \cdots \end{aligned} \tag{26}$$

expanded to the second order of small quantities.

6. *First Order Terms for* e_1.—We take the first order terms in (26), for which we require to know the values of P_2, P_2' and $\frac{\partial S}{\partial v_1}$; we have $P_2=\frac{3\cos^2\theta-1}{2}$, $P_2'=3\cos\theta$, and, from (20),

$$\begin{aligned}\frac{\partial S}{\partial v_1} = &-\cos(\omega+v)\sin(\omega_1+v_1)\cos(\phi-\phi_1)\\ &+\cos(\omega+v)\cos(\omega_1+v_1)\sin(\phi-\phi_1)\cos i_1\\ &+\sin(\omega+v)\sin(\omega_1+v_1)\sin(\phi-\phi_1)\cos i\\ &+\sin(\omega+v)\cos(\omega_1+v_1)[\cos i\cos i_1\cos(\phi-\phi_1)+\sin i\sin i_1]. \quad (27)\end{aligned}$$

In the first approximation we assume all the other elements of the binary orbit and all the elements of the hyperbolic orbit to remain constant; this is a legitimate approximation, since the effective encounter lasts for a time small when compared with the time of approach of m from a great distance and departure to a great distance. Nor is this invalidated when we integrate from $t=-\infty$ to $t=+\infty$, since the effect of m on the binary is negligible at large distances. We may therefore write, from (11),

$$\left.\begin{aligned} u_1 &= n_1 t+\chi_1, \qquad du_1 = n_1 dt\\ &= \frac{n\chi_1 - n_1\chi}{n}+\frac{n_1}{n}(e\sinh D - D)\\ &= A+\Delta, \text{ say},\\ e\cosh D\,dD &= n\,dt,\end{aligned}\right\} \quad (28)$$

and we may approximate for (12) by writing $v_1=u_1$ and $\cos v=\operatorname{sech} D$.

Taking the first order terms in (26) we have now

$$\frac{m_2^{\frac{2}{3}}(M+m)^{\frac{1}{3}}}{Mma_1^{\frac{2}{3}}f^{\frac{1}{3}}}\frac{de_1}{dD}=-\frac{\sin u_1}{\cosh^2 D}+\frac{3\cos\theta}{\cosh^2 D}\left[\sin u_1\cos\theta+2\cos u_1\frac{\partial S}{\partial v_1}\right]. \quad (29)$$

From (20) and (27) it is easy to see that $\cos\theta(\sin u_1\cos\theta+2\cos u_1\partial S/\partial v_1)$ can be expressed as a sum of sines and cosines of u_1 and $3u_1$. When we write $\Delta+A$ for u_1 in the right-hand side of (29) we shall, on integrating, have an expression for $[e_1]$, the increment of e_1, which is the sum of integrals of functions of D whose coefficients are $\sin A$, $\sin(A+2\omega)\cos^2(\phi-\phi_1)$, etc. If we suppose the stars in the neighbourhood of the binary to be fairly closely packed, so that such a large number of encounters takes place that all directions and distances are equally represented, then we may take average values over the ranges $0\leqslant\omega\leqslant 2\pi$, $0\leqslant\phi\leqslant 2\pi$, $-\pi/2\leqslant i\leqslant+\pi/2$ and $-\infty\leqslant A(\text{or }\chi)\leqslant+\infty$. It is easy to see that (29) is the sum of terms in sines and cosines of A and $3A$, without any terms independent of A, and the average values of all these four quantities over the range considered are zero. So that, to the first approximation the average value of $[e_1]$ is zero.

7. *First Order Terms for* a_1.—For the consideration of a_1 we begin with the equation

$$\frac{da_1}{dt}=\frac{2}{n_1a_1}k^2m\frac{\partial\Omega}{\partial\chi_1} \quad (30)$$

from (22),

$$=\frac{2k^2m}{n_1a_1}\frac{1}{1-e_1\cos u_1}\left[a_1e_1\sin u_1\frac{\partial\Omega}{\partial r_1}+\frac{(1-e_1^2)^{\frac{1}{2}}}{1-e_1\cos u_1}\frac{\partial\Omega}{\partial S}\frac{\partial S}{\partial v_1}\right]$$

from (14),

$$\frac{2k^2m}{n_1R^2}\left[\left\{2e_1\sin u_1P_2+(1-e_1^2)^{\frac{1}{2}}\frac{\partial S}{\partial v_1}P_2'\right\}\frac{a_1}{p}\frac{e\cos v+1}{e+1}\right.$$
$$\left.+(1-\beta)\left\{3e_1\sin u_1P_3+(1-e_1^2)^{\frac{1}{2}}\frac{\partial S}{\partial v_1}P_3'\right\}\left(\frac{a_1}{p}\right)^2(1-e_1\cos u_1)\left(\frac{e\cos v+1}{e+1}\right)^2+\ldots\right]. \quad (31)$$

Expanding this to the first two orders of small quantities we have

$$\frac{n_1p^2}{2k^2m}\frac{da_1}{dt}=\frac{\partial S}{\partial v_1}\frac{P_2'}{\cosh^3 D}\alpha_1-3\frac{\partial S}{\partial v_1}\frac{P_2'(\cosh D-1)}{\cosh^4 D}\alpha_1f+\frac{2\sin u_1P_2}{\cosh^3 D}\alpha_1e_1$$
$$-\frac{\cos u_1P_2'}{\cosh^3 D}\frac{\partial S}{\partial v_1}\alpha_1e_1+\frac{(1-\beta)P_3'}{\cosh^4 D}\frac{\partial S}{\partial v_1}\alpha_1^2. \quad (32)$$

We must write $n_1=\alpha_1^{-\frac{3}{2}}p^{-\frac{3}{2}}km_2^{\frac{3}{2}}/M$ and $a_1=\alpha_1p$ so that, taking only first order terms, we have

$$\frac{e^{\frac{1}{2}}m_2^{\frac{3}{2}}(M+m)^{\frac{1}{2}}}{3Mm}\frac{d\alpha_1^{-\frac{3}{2}}}{dD}=-\frac{3\cos\theta}{\cosh^2 D}\frac{\partial S}{\partial v_1}. \quad (33)$$

From this we see that, on taking an average over the ranges mentioned above, the average value of $[a_1]$ also is zero.

8. *Further Equations of Variations of Constants and the Values of their Increments at any Time.*—We must now proceed to the second approximations in which all the elements are allowed to vary; for this we shall need to know expressions for the increments $[\omega]$, $[\chi]$, etc. at any time. We obtain them, as above, using the well-known equations for the variations of constants. Let us begin with the other elements of the binary orbit.

For χ_1,

$$\frac{d\chi_1}{dt}=-\frac{1-e_1^2}{n_1a_1^2e_1}k^2m\frac{\partial\Omega}{\partial e_1}-\frac{2k^2m}{n_1a_1}\frac{\partial\Omega}{\partial a_1} \quad (34)$$

from (10),

$$=-\frac{k^2m}{n_1a_1}\left\{\frac{\partial\Omega}{\partial r_1}\left[\frac{1-e_1^2}{e_1}\left(-\cos u_1+e_1\sin u_1\frac{\partial u_1}{\partial e_1}\right)+2(1-e_1\cos u_1)\right]\right.$$
$$\left.+\frac{\partial\Omega}{\partial S}\frac{\partial S}{\partial v_1}\left[\frac{1-e_1^2}{a_1e_1}\frac{dv_1}{de_1}-\frac{3tn_1}{a_1}\frac{1}{(1-e_1\cos u_1)}\frac{\partial v_1}{\partial u_1}\right]\right\}.$$

From (11) and (12),

$$\frac{\partial u_1}{\partial e_1}=\frac{\sin u_1}{1-e_1\cos u_1},\qquad \frac{dv_1}{de_1}=\frac{(1-e_1^2)^{\frac{1}{2}}\sin u_1}{(1-e_1\cos u_1)^2}+\frac{\sin u_1}{(1-e_1^2)^{\frac{1}{2}}(1-e_1\cos u_1)},$$

so that, keeping first order terms only, we have

$$\frac{1}{}=\frac{mM\alpha_1^{\frac{3}{2}}f^{\frac{1}{2}}}{m_2^{\frac{3}{2}}(M+m)^{\frac{1}{2}}}\frac{1}{\cosh^2 D}\left\{\frac{2\cos u_1P_2}{e_1}+\left[\frac{3n_1}{n}(e\sinh D-D-\chi)-\frac{2\sin u_1}{e_1}\right]P_2'\frac{\partial S}{\partial v_1}\right\}. \quad (35)$$

If we assume $m_1 = m_2 = m/2$ we see that, on integrating (35) from $D = -\infty$ to $D = D$, $[\chi_1]$ is a small quantity of the first order and may be denoted by $\delta\chi_1$.

For ω_1,

$$\frac{d\omega_1}{dt} = \frac{-\cot i_1 k^2 m}{n_1 a_1^2 (1-e_1^2)^{\frac{1}{2}}} \frac{\partial\Omega}{\partial i_1} + \frac{(1-e_1^2)^{\frac{1}{2}}}{n_1 a_1^2 e_1} k^2 m \frac{\partial\Omega}{\partial e_1} \tag{36}$$

$$= \frac{k^2 m}{n_1 a_1^2 (1-e_1^2)^{\frac{1}{2}}} \left\{ -\cot i_1 \frac{\partial S}{\partial i_1} \frac{\partial\Omega}{\partial S} + \frac{1-e_1^2}{e_1} \left[a_1 \frac{\partial\Omega}{\partial r_1} \left(\frac{e_1 \sin^2 u_1}{1 - e_1 \cos u_1} - \cos u_1 \right) + \frac{\partial\Omega}{\partial S} \frac{\partial S}{\partial v_1} \frac{dv_1}{de_1} \right] \right\},$$

or, to first order, noticing that

$$\frac{\partial S}{\partial i_1} = -\cos(\omega + v) \sin(\omega_1 + v_1) \sin(\phi - \phi_1) \sin i_1 - \sin(\omega + v) \sin(\omega_1 + v_1) \cos i \sin i_1 \cos(\phi - \phi_1) + \sin(\omega + v) \sin(\omega_1 + v_1) \sin i \cos i_1$$

and neglecting terms small when compared with $\frac{1}{e_1}$,

$$\frac{d\omega_1}{dD} = -\frac{M m a_1^{\frac{3}{2}} f^{\frac{1}{2}}}{m_2^{\frac{3}{2}} (M+m)^{\frac{1}{2}}} \frac{1}{\cosh^2 D} \left\{ \frac{(3\cos^2\theta - 1)\cos u_1}{e_1} + \left[\frac{\sin(\omega + v) \sin(\omega_1 + v_1) \sin i \cos^2 i_1}{\sin i_1} - \frac{2 \sin u_1}{e_1} \frac{\partial S}{\partial v_1} \right] \times 3 \cos\theta \right\}. \tag{37}$$

On integrating we see that $[\omega_1] \equiv \delta\omega_1$ is a small quantity of the first order plus cosec $i_1 \times$ a small quantity of the second order.

For ϕ_1,

$$\frac{d\phi_1}{dt} = \frac{\operatorname{cosec} i_1 k^2 m}{n_1 a_1^2 (1-e_1^2)^{\frac{1}{2}}} \frac{\partial\Omega}{\partial i_1}. \tag{38}$$

Using the same process as above with ω_1, we have, to the first order,

$$\frac{d\phi_1}{dD} = \frac{3 m M a_1^{\frac{3}{2}} f^{\frac{1}{2}}}{m_2^{\frac{3}{2}} (M+m)^{\frac{1}{2}}} \frac{\sin(\omega + v) \sin(\omega_1 + v_1) \sin i \cos i_1 \cos\theta}{\cosh^2 D \sin i_1}. \tag{39}$$

We see that $\delta\phi_1$ is a small quantity of the second order $\times$ cosec i_1, and we notice that $\delta\omega_1 + \cos i_1 \delta\phi_1$ is a small quantity of the first order whatever the value of i_1.

For i_1,

$$\frac{di_1}{dt} = \frac{\cot i_1 k^2 m}{n_1 a_1^2 (1-e_1^2)^{\frac{1}{2}}} \frac{\partial\Omega}{\partial\omega_1} - \frac{\operatorname{cosec} i_1}{n_1 a_1^2 (1-e_1^2)^{\frac{1}{2}}} k^2 m \frac{\partial\Omega}{\partial\phi_1} = \frac{k^2 m}{n_1 a_1^2 (1-e_1^2)^{\frac{1}{2}}} \frac{1}{\sin i_1} \frac{\partial\Omega}{\partial S} \left[\cos i_1 \frac{\partial S}{\partial\omega_1} - \frac{\partial S}{\partial\phi_1} \right]. \tag{40}$$

Now

$$\cos i_1 \frac{\partial S}{\partial\omega_1} - \frac{\partial S}{\partial\phi_1} = -\cos(\omega + v) \cos(\omega_1 + v_1) \sin(\phi - \phi_1) \sin^2 i_1 - \sin(\omega + v) \cos(\omega_1 + v_1) \cos(\phi - \phi_1) \times \cos i \sin^2 i_1 + \sin(\omega + v) \cos(\omega_1 + v_1) \sin i \cos i_1 \sin i_1,$$

so that to the first order,

$$\frac{di_1}{dD}=\frac{3Mma_1^{\frac{3}{2}}f^{\frac{1}{2}}}{m_2^{\frac{3}{2}}(M+m)^{\frac{1}{2}}}\frac{\cos\theta}{\cosh^2 D}\{\sin(\omega+v)\cos(\omega_1+v_1)\sin i\cos i_1 - \sin i_1\cos(\omega_1+v_1)[\cos(\omega+v)\sin(\phi-\phi_1)+\sin(\omega+v)\times\cos(\phi-\phi_1)\cos i]\}. \quad (41)$$

From this we see that δi_1 is of the second order of small quantities. On evaluating the increments of the elements for the hyperbolic orbit, it is found that δe and $\delta\chi$ are of the second order, $\delta a/a$, $\delta\omega+\cos i\delta\phi$, and δi are of the third order of small quantities; also we have seen δe_1 and $\delta a_1/a_1$ to be of the second order of small quantities. It will now be useful to evaluate $\sin\theta\delta\theta$ as far as it contains first order increments.

Differentiating equation (20) we have

$$-\sin\theta\delta\theta=\frac{\partial S}{\partial\omega}(\delta\omega+\delta v)+\frac{\partial S}{\partial\omega_1}(\delta\omega_1+\delta v_1)+\frac{\partial S}{\partial\phi_1}(\delta\phi_1-\delta\phi)+\frac{\partial S}{\partial i_1}\delta i_1+\frac{\partial S}{\partial i}\delta i. \quad (42)$$

We have $\delta u_1=(\sin u_1\delta e_1+\delta\chi_1+t\delta n_1)/(1-e_1\cos u_1)$, so that

$$\delta v_1=\frac{(1-e_1^2)^{\frac{1}{2}}}{(1-e_1\cos u_1)^2}(\delta\chi_1+t\delta n_1)+\frac{2-e_1\cos u_1-e_1^2}{(1-e_1^2)^{\frac{1}{2}}(1-e_1\cos u_1)^2}\sin u_1\delta e_1.$$

With the help of this and substituting in (42) from (35), (37) and (39) we have

$$-\sin\theta\delta\theta=K\Bigg\{-\frac{\partial S}{\partial\omega_1}\int_{-\infty}^{D}\left[\frac{(3\cos^2\theta-1)\cos u_1}{e_1}-\frac{2\sin u_1}{e_1}\frac{\partial S}{\partial\omega_1}3\cos\theta\right]\frac{dD}{\cosh^2 D} + \frac{\partial S}{\partial\omega_1}\frac{(1-e_1^2)^{\frac{1}{2}}}{(1-e_1\cos u_1)^2}\int_{-\infty}^{D}\left[\frac{(3\cos^2\theta-1)\cos u_1}{e_1} + 3\cos\theta\frac{\partial S}{\partial\omega_1}\left(3\Delta-\frac{3n_1\chi}{n}-\frac{2\sin u_1}{e_1}\right)\right]\frac{dD}{\cosh^2 D}\Bigg\},$$

where $K=\dfrac{Mma_1^{\frac{3}{2}}f^{\frac{1}{2}}}{m_2^{\frac{3}{2}}(M+m)^{\frac{1}{2}}}$. The term in $\operatorname{cosec} i_1$ in $\delta\omega_1$ and $\delta\phi_1$ has been omitted, since its coefficient would be $\cos i_1\dfrac{\partial S}{\partial\omega_1}-\dfrac{\partial S}{\partial\phi_1}$, which, as we have seen, is an expression with a factor $\sin i_1$. We have finally, to the first order,

$$-\sin\theta\delta\theta=\frac{9Mma_1^{\frac{3}{2}}f^{\frac{1}{2}}}{m_2^{\frac{3}{2}}(M+m)^{\frac{1}{2}}}\frac{\partial S}{\partial\omega_1}\int_{-\infty}^{D}\cos\theta\frac{\partial S}{\partial\omega_1}\left(\Delta-\frac{n_1\chi}{n}\right)\frac{dD}{\cosh^2 D}. \quad (43)$$

In a similar manner we find that, from (27),

$$\delta\left(\frac{\partial S}{\partial\omega_1}\right)=\frac{9Mma_1^{\frac{3}{2}}f^{\frac{1}{2}}}{m_2^{\frac{3}{2}}(M+m)^{\frac{1}{2}}}\frac{\partial^2 S}{\partial\omega_1^2}\int_{-\infty}^{D}\left(\Delta-\frac{n_1\chi}{n}\right)\cos\theta\frac{\partial S}{\partial\omega_1}\frac{dD}{\cosh^2 D}. \quad (44)$$

9. *Second Approximation for* $[e_1]$.—The second approximation consists in allowing the elements to vary in the first-order terms, but not in the second-order terms; these last will contribute to $[e_1]$ expressions whose average values vanish when taken over the prescribed ranges. In equation

(29) we shall write $e \equiv e + \delta e$, etc., where the first term is to be regarded as constant and at its value at the beginning of the encounter. Having regard only to the first order increments we have at once,

$$\frac{(M+m)^{\frac{1}{2}} m_2^{\frac{3}{2}}}{2mM\alpha_1^{\frac{3}{2}} f^{\frac{1}{2}}} \frac{de_1}{dD} = \frac{1}{\cosh^2 D}\left[\frac{\sin u_1(3\cos^2\theta - 1)}{2} + 3\cos\theta \frac{\partial S}{\partial \omega_1}\cos u_1\right]$$
$$- 3\left(\sin u_1 \cos\theta + \cos u_1 \frac{\partial S}{\partial \omega_1}\right)\frac{\sin\theta\delta\theta}{\cosh^2 D} + \frac{3\cos\theta\cos u_1}{\cosh^2 D}\delta\left(\frac{\partial S}{\partial \omega_1}\right)$$
$$+ \left(\frac{3\cos^2\theta - 1}{2}\cos u_1 - 3\cos\theta \frac{\partial S}{\partial \omega_1}\sin u_1\right)\frac{\delta\chi_1}{\cosh^2 D}. \qquad (45)$$

Since we are concerned only with the average of δe_1 for encounters in all directions, we may neglect the terms on the right of (45) independent of the increments, and write from (35), (43) and (44),

$$\frac{m_2^3(M+m)}{2M^2m^2\alpha_1^3 f}\frac{de_1}{dD} = \left(\frac{3\cos^2\theta - 1}{2}\cos u_1 - 3\cos\theta\sin u_1\frac{\partial S}{\partial \omega_1}\right)\frac{2}{\cosh^2 D}$$
$$\times \int_{-\infty}^{D}\left(\frac{\cos u_1 P_2}{e_1} - \frac{\sin u_1}{e_1}P_2'\frac{\partial S}{\partial \omega_1}\right)\frac{dD}{\cosh^2 D}$$
$$+ \left[\frac{3\cos^2\theta - 1}{2}\cos u_1 - 3\cos\theta\sin u_1\frac{\partial S}{\partial \omega_1}\right.$$
$$\left. + 3\left(\sin u_1\cos\theta + \cos u_1\frac{\partial S}{\partial \omega_1}\right)\frac{\partial S}{\partial \omega_1} + 3\cos\theta\cos u_1\frac{\partial^2 S}{\partial \omega_1^2}\right]$$
$$\times \frac{3}{\cosh^2 D}\int_{-\infty}^{D}\frac{\Delta - \frac{n_1\chi}{n}}{\cosh^2 D}P_2'\frac{\partial S}{\partial \omega_1}dD. \qquad (46)$$

We shall approximate by putting $v_1 = u_1$, $\cos v = \operatorname{sech} D$, $\sin v = \tanh D$. There can be no loss of generality in assuming $\omega_1 = 0$, $i_1 = \pi/2$, so long as these substitutions are made after differentiation, since we may choose our axes in any direction. So we have from (20),

$$4\cos^2\theta = [1 + \cos 2(\omega+v)](1+\cos 2u_1)\cos^2(\phi-\phi_1)$$
$$+ [1 - \cos 2(\omega+v)][(1+\cos 2u_1)\sin^2(\phi-\phi_1)\cos^2 i$$
$$+ (1 - \cos 2u_1)\sin^2 i - 2\sin 2u_1 \sin(\phi-\phi_1)\cos i \sin i]$$
$$+ 2\sin 2(\omega+v)[\sin 2u_1\cos(\phi-\phi_1)\sin i$$
$$- (1+\cos 2u_1)\sin(\phi-\phi_1)\cos(\phi-\phi_1)\cos i]$$
$$= \cos^2(\phi-\phi_1) + \sin^2 i + \sin^2(\phi-\phi_1)\cos^2 i$$
$$- 2\sin 2(\omega+v)\sin(\phi-\phi_1)\cos(\phi-\phi_1)\cos i + \cos 2(\omega+v)$$
$$\times [\cos^2(\phi-\phi_1) - \sin^2 i - \cos^2 i \sin^2(\phi-\phi_1)]$$
$$+ \cos 2u_1[\cos^2(\phi-\phi_1) - \sin^2 i + \sin^2(\phi-\phi_1)\cos^2 i$$
$$- \sin 2(\omega+v)\sin(\phi-\phi_1)\cos(\phi-\phi_1)\cos i + \cos 2(\omega+v)$$
$$\times \{\cos^2(\phi-\phi_1) + \sin^2 i - \cos^2 i \sin^2(\phi-\phi_1)\}]$$
$$- 2\sin 2u_1[\sin(\phi-\phi_1)\cos i \sin i - \sin 2(\omega+v)\cos(\phi-\phi_1)\sin i$$
$$- \cos 2(\omega+v)\sin(\phi-\phi_1)\cos i \sin i]. \qquad (47)$$

By differentiation,

$$4\cos\theta\frac{\partial S}{\partial\omega_1} = -2\cos 2u_1[\sin(\phi-\phi_1)\cos i\sin i - \text{etc.}] - \sin 2u_1[\cos^2(\phi-\phi_1) - \text{etc.}]. \quad (48)$$

The first bracket in the first term on the right-hand side of (46) will be therefore

$$\begin{aligned}-\frac{3\cos u_1}{2} &+ \tfrac{3}{8}\cos u_1[\cos^2(\phi-\phi_1) + \sin^2 i + \sin^2(\phi-\phi_1)\cos^2 i \\ &- 2\sin 2(\omega+v)\sin(\phi-\phi_1)\cos(\phi-\phi_1)\cos i + \cos 2(\omega+v) \\ &\times\{\cos^2(\phi-\phi_1) - \sin^2 i - \cos^2 i\sin^2(\phi-\phi_1)\}] \\ &+ \tfrac{3}{16}(3\cos u_1 - \cos 3u_1)[\cos^2(\phi-\phi_1) - \text{etc.}] \\ &- \tfrac{3}{8}(3\sin u_1 - \sin 3u_1)[\sin(\phi-\phi_1)\cos i\sin i - \text{etc.}]. \quad (49)\end{aligned}$$

The integrand in the first term on the right of (46) is $(49)/e_1$. Remembering that $u_1 = \Delta + A$, we see that (49) is a series in sines and cosines of A, $A \pm 2\omega$ and $3A \pm 2\omega$; on taking an average for all values of ω from 0 to 2π and for all values of A (or χ) we find that the averages of such terms as $\cos A \sin(A+2\omega)$ are zero, the only terms yielding any other results being $\cos A\cos A$, etc., whose averages are 0·5. The coefficient of $\cos A$ in (49) is

$$\begin{aligned}&\tfrac{3}{8}\cos\Delta[-4+\cos^2(\phi-\phi_1)+\sin^2 i+\sin^2(\phi-\phi_1)\cos^2 i] \\ &\quad+\tfrac{9}{16}[\cos\Delta\{\cos^2(\phi-\phi_1)-\sin^2 i+\sin^2(\phi-\phi_1)\cos^2 i\} \\ &\qquad -2\sin\Delta\sin(\phi-\phi_1)\cos i\sin i] \\ &=\tfrac{3}{16}\cos\Delta[-8+5\cos^2(\phi-\phi_1)-\sin^2 i+5\sin^2(\phi-\phi_1)\cos^2 i] \\ &\qquad -\tfrac{9}{8}\sin\Delta\sin(\phi-\phi_1)\sin i\cos i.\end{aligned}$$

Consider $\int_{-\infty}^{\infty} f(D)\int_{-\infty}^{D} f(D)dDdD$, when f is an odd function, so that $\int f(D)dD$ is an even function; then $\int_{-\infty}^{\infty} f(D)\int_{-\infty}^{D} f(D)dDdD = \int_{-\infty}^{\infty}(\text{odd function})dD = 0$.

If f is an even function,

$$\int_{-\infty}^{D} f(D)dD = F(D) - F(-\infty),$$

where F is an odd function

$$= F(D) + F(\infty);$$

now

$$\begin{aligned}\int_{-\infty}^{\infty} f(D)\int_{-\infty}^{D} f(D)dDdD &= \int_{-\infty}^{\infty}(\text{odd function})\,dD + F(\infty)\int_{-\infty}^{\infty} f(D)dD \\ &= 2\left[\int_0^{\infty} f(D)dD\right]^2.\end{aligned}$$

The average value of $[5\cos^2(\phi-\phi_1)+5\sin^2(\phi-\phi_1)\cos^2 i - \sin^2 i - 8]^2$ over all $\phi-\phi_1$ from 0 to 2π and all i from $-\pi/2$ to $+\pi/2$ is

$$25(\tfrac{3}{8}+\tfrac{3}{8}\cdot\tfrac{3}{8})+\tfrac{3}{8}+64+50\cdot\tfrac{1}{2}\cdot\tfrac{1}{8}-10\cdot\tfrac{1}{2}\cdot\tfrac{1}{2}-80\cdot\tfrac{1}{2}-10\cdot\tfrac{1}{2}\cdot\tfrac{1}{8}+16\cdot\tfrac{1}{2}-80\cdot\tfrac{1}{2}\tfrac{1}{2} = 25\tfrac{17}{64}.$$

The average of $[5\cos^2(\phi-\phi_1)+\text{ etc.}]\sin(\phi-\phi_1)\sin i\cos i$ is 0.
The coefficient of $\sin A$ in (49) is

$$-\tfrac{9}{8}\cos\Delta\sin(\phi-\phi_1)\sin i\cos i-\tfrac{3}{16}\sin\Delta[-8+\text{ etc.}].$$

The average of $\sin^2(\phi-\phi_1)\sin^2 i\cos^2 i$ is $\frac{1}{16}$; so that the average contribution of the A terms to the integrated first term on the right of (46) is

$$\frac{15849}{8192e_1}\left[\int_0^\infty\frac{\cos\Delta dD}{\cosh^2 D}\right]^2. \tag{50}$$

The coefficient of $\cos(A+2\omega)$ in (49) is

$$\tfrac{3}{32}\cos(\Delta+2v)[5\cos^2(\phi-\phi_1)+\sin^2 i+\cos^2 i\sin^2(\phi-\phi_1)-2\sin i\cos(\phi-\phi_1)]$$
$$-\tfrac{3}{16}\sin(\Delta+2v)[5\cos(\phi-\phi_1)\sin(\phi-\phi_1)\cos i-3\sin(\phi-\phi_1)\cos i\sin i],$$

and that of $\sin(A+2\omega)$ is

$$\tfrac{3}{16}\sin(\phi-\phi_1)\cos i[3\sin i-5\cos(\phi-\phi_1)]\cos(\Delta+2v)$$
$$-\tfrac{3}{32}\sin(\Delta+2v)[5\cos^2(\phi-\phi_1)+\text{ etc.}].$$

The average of

$$[5\cos^2(\phi-\phi_1)+\sin^2 i+\cos^2 i\sin^2(\phi-\phi_1)-2\sin i\cos(\phi-\phi_1)]^2 \text{ is } \tfrac{905}{64},$$

and that of

$$[3\sin(\phi-\phi_1)\cos i\sin i-5\sin(\phi-\phi_1)\cos(\phi-\phi_1)\cos i]^2 \text{ is } \tfrac{17}{8},$$

so that the average contribution of $(A+2\omega)$ terms is

$$\frac{13041}{32768e_1}\left[\int_0^\infty\frac{\cos(\Delta+2v)}{\cosh^2 D}dD\right]^2. \tag{51}$$

In the same way it is easy to see that the average contribution of the $(A-2\omega)$ terms to the integrated first term on the right of (46) is

$$\frac{13041}{32768e_1}\left[\int_0^\infty\frac{\cos(\Delta-2v)}{\cosh^2 D}dD\right]^2. \tag{52}$$

The coefficient of $\cos 3A$ in (49) is

$$-\tfrac{3}{16}[\cos 3\Delta\{\cos^2(\phi-\phi_1)-\sin^2 i+\sin^2(\phi-\phi_1)\cos^2 i\}-2\sin 3\Delta\sin(\phi-\phi_1)\cos i\sin i]$$

and that of $\sin 3A$ is

$$\tfrac{3}{16}[2\cos 3\Delta\sin(\phi-\phi_1)\cos i\sin i+\sin 3\Delta\{\cos^2(\phi-\phi_1)+\sin^2(\phi-\phi_1)\cos^2 i-\sin^2 i\}]$$

The average of

$$\sin^2(\phi-\phi_1)\cos^2 i\sin^2 i \quad \text{is} \quad \tfrac{1}{16},$$

and that of

$$[\cos^2(\phi-\phi_1)-\sin^2 i+\sin^2(\phi-\phi_1)\cos^2 i] \quad \text{is} \quad \tfrac{25}{64},$$

so that the contribution of the $3A$ terms is

$$\frac{369}{8192e_1}\left[\int_0^\infty\frac{\cos 3\Delta dD}{\cosh^2 D}\right]^2. \tag{53}$$

The coefficient of $\cos(3A+2\omega)$ in (49) is

$$\tfrac{3}{32}\cos(3\Delta+2v)[\cos^2(\phi-\phi_1)+\sin^2 i+\cos^2 i\sin^2(\phi-\phi_1)-2\cos(\phi-\phi_1)\sin i]$$
$$+\tfrac{3}{16}\sin(3\Delta+2v)[\sin(\phi-\phi_1)\cos i\sin i-\sin(\phi-\phi_1)\cos(\phi-\phi_1)\cos i],$$

and that of $\sin(3A+2\omega)$ is

$$\tfrac{3}{16}\cos(3\Delta+2v)\sin(\phi-\phi_1)\cos i[\sin i-\cos(\phi-\phi_1)]-\tfrac{3}{32}\sin(3\Delta+2v)[\cos^2(\phi-\phi_1)+\text{etc.}].$$

The average of

$$[\cos^2(\phi-\phi_1)+\sin^2 i+\cos^2 i\sin^2(\phi-\phi_1)-2\sin i\cos(\phi-\phi_1)]^2 \quad \text{is} \quad \tfrac{169}{64},$$

and that of

$$[\sin(\phi-\phi_1)\cos i\sin i-\sin(\phi-\phi_1)\cos(\phi-\phi_1)\cos i]^2 \quad \text{is} \quad \tfrac{1}{8},$$

so that the contribution of the $(3A+2\omega)$ terms is on the average

$$\frac{1809}{32768e_1}\left[\int_0^\infty \frac{\cos(3\Delta+2v)}{\cosh^2 D}dD\right]^2. \tag{54}$$

Similarly, the contribution of the $(3A-2\omega)$ terms is

$$\frac{1809}{32768e_1}\left[\int_0^\infty \frac{\cos(3\Delta-2v)}{\cosh^2 D}dD\right]^2. \tag{55}$$

We must now consider the second term in (46); it is easy to see that the coefficient of the integral is a series of sines and cosines of A, $A\pm2\omega$, $3A$, $3A\pm2\omega$, and that $P_2'\partial S/\partial\omega_1$ is a series of sines and cosines of $2A$, $2A\pm2\omega$. It follows that the Δ terms in the integrand will yield nothing on the average, for the average value of any of the products $\sin A\cos(2A-2\omega)$ is zero. Let us suppose χ_1 to be zero, a legitimate assumption, since we can choose our time origin anywhere. We shall then be left with terms in $A\sin 2A\cos(A-2\omega)$ and similar expressions, of which the only ones yielding any average results other than zero are those in $A\sin A$, $A\sin 3A$ and $A\sin 5A$. Let us consider the average of $A\sin pA$; it is

$$\left[\frac{1}{A}\int_0^A A\sin pA\,dA\right]_{A=\infty}=\left[\frac{1}{A}\left(-\frac{A}{p}\cos pA+\frac{1}{p^2}\sin pA\right)\right]_{A=\infty}$$
$$=-\frac{1}{p}[\cos pA]_{A=\infty}.$$

It is possible to choose the upper limit of χ such that $A=(2q+1)\pi/2$, where q is a very large integer. In this case we see that the χ terms yield no average result to (46), and we may write down the final result from (50), (51), (52), (53), (54) and (55).

$$[e_1]=\frac{m^2M^2}{m_2^3(M+m)}\frac{a_1^3 f}{e_1}\left\{\frac{15849}{4096}\left[\int_0^\infty\frac{\cos\Delta}{\cosh^2 D}dD\right]^2+\frac{369}{4096}\left[\int_0^\infty\frac{\cos 3\Delta dD}{\cosh^2 D}\right]^2\right.$$
$$+\frac{13041}{8192}\left[\left(\int_0^\infty\frac{\cos\Delta(1-\sinh^2 D)}{\cosh^4 D}dD\right)^2+\left(\int_0^\infty\frac{2\sinh D\sin\Delta dD}{\cosh^4 D}\right)^2\right]$$
$$\left.+\frac{1809}{8192}\left[\left(\int_0^\infty\frac{\cos 3\Delta(1-\sinh^2 D)}{\cosh^4 D}dD\right)^2+\left(\int_0^\infty\frac{2\sinh D\sin 3\Delta}{\cosh^4 D}dD\right)^2\right]\right\}. \tag{56}$$

10. *Second Approximation for* $[a_1]$.—We now return to the consideration of a_1 which we shall treat in a similar manner. Take the equation (33) and proceed as before, substituting from (43) and (44) so that

$$\frac{1}{54}\frac{(M+m)m_2^3}{M^2m^2a_1{}^4f}\frac{da_1}{dD}=\frac{\cos\theta\dfrac{\partial^2S}{\partial\omega_1{}^2}+\left(\dfrac{\partial S}{\partial\omega_1}\right)^2}{\cosh^2 D}\int_{-\infty}^{D}\frac{\cos\theta\dfrac{\partial S}{\partial\omega_1}}{\cosh^2 D}\left(\Delta-\frac{n_1\chi}{n}\right)dD. \quad (57)$$

By differentiating (48) we have

$$2\left[\cos\theta\frac{\partial^2S}{\partial\omega_1{}^2}+\left(\frac{\partial S}{\partial\omega_1}\right)^2\right]=-\cos 2u_1[\cos^2(\phi-\phi_1)-\text{ etc.}]$$
$$+2\sin 2u_1[\sin(\phi-\phi_1)\cos i\sin i-\text{ etc.}]. \quad (58)$$

The coefficient of $\cos 2A$ in (58) is

$$-\cos 2\Delta[\cos^2(\phi-\phi_1)+\sin^2(\phi-\phi_1)\cos^2 i-\sin^2 i]$$
$$+2\sin 2\Delta\sin(\phi-\phi_1)\cos i\sin i.$$

The coefficient of $\cos 2A$ in $4\Delta\cos\theta\dfrac{\partial S}{\partial\omega_1}$ is

$$-\Delta\sin 2\Delta[\cos^2(\phi-\phi_1)+\sin^2(\phi-\phi_1)\cos^2 i-\sin^2 i]$$
$$-2\Delta\cos 2\Delta\sin(\phi-\phi_1)\cos i\sin i.$$

The coefficients of $\sin 2A$ are respectively

$$\sin 2\Delta[\cos^2(\phi-\phi_1)+\sin^2(\phi-\phi_1)\cos^2 i-\sin^2 i]$$
$$+2\cos 2\Delta\sin(\phi-\phi_1)\cos i\sin i$$

and

$$-\Delta\cos 2\Delta[\cos^2(\phi-\phi_1)+\sin^2(\phi-\phi_1)\cos^2 i-\sin^2 i]$$
$$+2\Delta\sin 2\Delta\sin(\phi-\phi_1)\cos i\sin i.$$

The contribution of the $(2A, \Delta)$ terms to the integrated right-hand side of (57) is on the average

$$\frac{41}{512}\int_0^\infty\frac{\cos 2\Delta}{\cosh^2 D}dD\int_0^\infty\frac{\Delta\sin 2\Delta}{\cosh^2 D}dD. \quad (59)$$

The coefficient of $\cos(2A+2\omega)$ in (58) is

$$-\tfrac{1}{2}\cos 2(\Delta+v)[\cos^2(\phi-\phi_1)+\sin^2 i-\sin^2(\phi-\phi_1)\cos^2 i-2\cos(\phi-\phi_1)\sin i]$$
$$+\sin 2(\Delta+v)\sin(\phi-\phi_1)\cos i[\cos(\phi-\phi_1)-\sin i]$$

and in $4\Delta\cos\theta\dfrac{\partial S}{\partial\omega_1}$ it is

$$-\tfrac{1}{2}\Delta\sin 2(\Delta+v)[\cos^2(\phi-\phi_1)+\sin^2 i-\sin^2(\phi-\phi_1)\cos^2 i-2\cos(\phi-\phi_1)\sin i]$$
$$-\Delta\cos 2(\Delta+v)\sin(\phi-\phi_1)\cos i[\cos(\phi-\phi_1)-\sin i].$$

The coefficient of $\sin 2(A+\omega)$ in (58) is

$$\cos 2(\Delta+v)\sin(\phi-\phi_1)\cos i[\cos(\phi-\phi_1)-\sin i]$$
$$+\tfrac{1}{2}\sin 2(\Delta+v)[\cos^2(\phi-\phi_1)+\text{ etc.}]$$

and in $4\Delta \cos\theta \frac{\partial S}{\partial \omega_1}$ it is

$$\Delta \sin 2(\Delta+v)\sin(\phi-\phi_1)\cos i[\cos(\phi-\phi_1)-\sin i]$$
$$-\tfrac{1}{2}\Delta\cos 2(\Delta+v)[\cos^2(\phi-\phi_1)+\text{ etc.}].$$

So the average contribution of the $[(2A+2\omega), \Delta]$ terms is

$$\frac{169}{2048}\int_0^\infty \frac{\cos 2(\Delta+v)}{\cosh^2 D}dD\int_0^\infty \frac{\Delta\sin 2(\Delta+v)}{\cosh^2 D}dD. \tag{60}$$

Similarly the average contribution of the $[(2A-2\omega), \Delta]$ terms is

$$\frac{169}{2048}\int_0^\infty \frac{\cos 2(\Delta-v)}{\cosh^2 D}dD\int_0^\infty \frac{\Delta\sin 2(\Delta-v)}{\cosh^2 D}dD. \tag{61}$$

Now we must return to the χ terms. Assuming, as before, that $\chi_1=0$, we have left only terms in $A\sin 2A\cos 2A$ whose average is

$$\tfrac{1}{2}\left[\frac{1}{A}\int_0^A A\sin 4A\right]_{A=\infty} = \tfrac{1}{2}\left[\frac{1}{A}\left(-\frac{A}{4}\cos 4A+\frac{\sin 4A}{16}\right)\right]_{A=\infty}$$
$$= -\tfrac{1}{8}[\cos 4A]_{A=\infty}$$
$$=0 \text{ for } A=(2p+1)\pi/8,$$

and terms in $A\sin^2 2A$, $A\cos^2 2A$ whose averages are zero. Thus the χ terms can be made to vanish and we are left with

$$\frac{\delta a_1}{a_1}=\frac{27M^2m^2}{m_2{}^3(M+m)}a_1{}^3 f\left\{\frac{41}{256}\int_0^\infty \frac{\cos 2\Delta dD}{\cosh^2 D}\int_0^\infty \frac{\Delta\sin 2\Delta}{\cosh^2 D}dD\right.$$
$$+\frac{169}{512}\left[\int_0^\infty \frac{\cos 2\Delta(1-\sinh^2 D)}{\cosh^4 D}dD\int_0^\infty \frac{(1-\sinh D^2)\Delta\sin 2\Delta}{\cosh^4 D}dD\right.$$
$$\left.\left.-\int_0^\infty \frac{2\sin 2\Delta\sinh D}{\cosh^4 D}dD\int_0^\infty \frac{2\sinh D\Delta\cos 2\Delta}{\cosh^4 D}dD\right]\right\}. \tag{62}$$

11. *Numerical Values for* $[e_1]$ *and* $[a_1]$.—We shall require to know the number of stars which pass the binary at the distance p with orbits of eccentricity e. Jeans (*Astronomy and Cosmogony*, p. 310) shows that the number of stars passing in unit time with velocity V_0 and original distance of straight path g is $\pi\nu V_0 g^2$ where ν is the density of stars in the neighbourhood ; so the number in the range V_0 to V_0+dV_0 and g to $g+dg$ is

$$2\pi\nu g dV_0 dg. \tag{63}$$

We have $$g^2=a^2(e^2-1)=p^2(e+1)/(e-1)$$

or $$g=p\left(1+\frac{1}{e}+\ldots\right);$$

$$V_0{}^2=\frac{2k^2M^3}{(M+m)^2}\frac{a(e-1)}{a^2(e^2-2)}$$
$$=\frac{2k^2M^3}{(M+m)^2}\frac{1}{p}\frac{(e-1)^2}{e^2-2},$$

or
$$V_0 = \frac{\sqrt{2}kM^{\frac{3}{2}}}{M+m}\frac{1}{p^{\frac{1}{2}}}\left(1-\frac{1}{e}+\ldots\right).$$

$$\frac{\partial(V_0, g)}{\partial(p, e)} \text{ is therefore } \frac{\sqrt{2}kM^{\frac{3}{2}}}{M+m}\left\{-\frac{\frac{1}{2}\left(1-\frac{1}{e}\right)}{p^{\frac{3}{2}}}\left(-\frac{p}{e^2}\right)-\frac{1}{p^{\frac{1}{2}}e^2}\left(1+\frac{1}{e}\right)\right\}$$

$$= -\frac{\sqrt{2}\,kM^{\frac{3}{2}}}{2(M+m)}\frac{1+\frac{3}{e}}{p^{\frac{1}{2}}e^2}.$$

So the number of stars passing in the range e to $e+de$, p to $p+dp$ is
$$\frac{\sqrt{2}kM^{\frac{3}{2}}}{M+m}\pi\nu T\frac{p^{\frac{1}{2}}dpde}{e^2}$$
to the first order, where T is the total time under consideration. Let
$$h = 1/a_1 = p/a_1,$$
so that
$$a_1 dh = dp,$$
and put
$$k = n_1 a_1^{\frac{3}{2}} M/m_2^{\frac{3}{2}} = 2\pi M a_1^{\frac{3}{2}}/m_2^{\frac{3}{2}} T_1,$$
where T_1 is the period of the binary ; then we can write the above expression, with $m_1 = m_2 = m/2$, as
$$4\pi^2\nu a_1^3\frac{T}{T_1}\frac{h^{\frac{1}{2}}dhde}{e^2}. \tag{64}$$

The integrals in expressions (56) and (62) do not seem to be integrable ; I have drawn graphs of the functions

$\cos\Delta(1-\sinh^2 D)/\cosh^4 D$, $2\sinh D\sin\Delta/\cosh^4 D$,
$\cos\Delta/\cosh^2 D$, $\sin\Delta/\cosh^2 D$, $\Delta\sin\Delta(1-\sinh^2 D)/\cosh^4 D$,
$2\sinh D\,\Delta\cos\Delta/\cosh^4 D$ and $\Delta\sin\Delta/\cosh^2 D$

for different values of h^3/e. If the corresponding integrals are denoted respectively by $I_1, I_2 \ldots I_7$ we have the following table :—

Table I

h^3/e	I_1	I_2	I_3	I_4	I_5	I_6	I_7
0·01	0·3312	0·0280	0·9840	0·0280	−0·0009	−0·0081	0·0011
1	0·4126	0·1780	0·9186	0·2280	0·0452	0·0694	−0·0327
100	0·2840	0·2834	0·1490	0·4080	0·1800	−0·1720	0·0800

For $h^3/e = 10{,}000$ all the integrals are negligible ; also for any fixed h, as e increases and
$$\Delta \equiv h^{\frac{3}{2}}(\sinh D - D/e)/4e^{\frac{1}{2}} \to 0, \qquad I_1 \to 1/3, \qquad I_3 \to 1,$$

and the other integrals tend to zero. Let us suppose the integrals have constant values, as shown in Table II, over certain ranges of h^3/e.

TABLE II

	I_1	I_2	I_3	I_4	I_5	I_6	I_7
$e \geqslant 100h^3$	0·33	0	1	0	0	0	0
$100h^3 \geqslant e \geqslant h^3$	0·37	0·1	0·95	0·15	0·03	0·05	−0·02
$h^3 \geqslant e \geqslant 0{\cdot}01h^3$	0·30	0·25	0·30	0·37	0·15	−0·15	0·05
$0{\cdot}01h^3 \geqslant e \geqslant 0{\cdot}0001h^3$	0·10	0·10	0·05	0·15	0·06	−0·05	0·03
$0{\cdot}0001h^3 \geqslant e$	0	0	0	0	0	0	0

To adjust for a factor 2 or 3 before the Δ, we take the limits as $400h^3$ or $900h^3$, $4h^3$ or $9h^3$, etc., respectively.

From (56) we now have

$$[e_1] = \frac{16\pi^2 \nu a_1{}^3}{e_1}\frac{T}{T_1}\int_5^\infty\int_5^\infty \frac{1}{e^3 h^{\frac{5}{2}}}\left\{\frac{15849}{4096}\left[\int_0^\infty \frac{\cos\Delta}{\cosh^2 D}dD\right]^2 + \text{etc.}\right\}dhde.$$

Let us evaluate

$$\int_5^\infty\int_5^\infty\left[\int_0^\infty \frac{\cos\Delta(1-\sinh^2 D)}{\cosh^4 D}dD\right]^2\frac{dhde}{e^3 h^{\frac{5}{2}}};$$

we must divide the range $0{\cdot}0001h^3 \leqslant e \leqslant 0{\cdot}01h^3$ into three parts, $h < 5\times 2^{\frac{2}{3}}$ in which we do not integrate, $5\times 2^{\frac{2}{3}} \leqslant h < 10\times 50^{\frac{1}{3}}$ in which we integrate from $e = 5$ to $e = 0{\cdot}01h^3$, and $h \geqslant 10\times 50^{\frac{1}{3}}$ in which we integrate from $e = 0{\cdot}0001h^3$ to $e = 0{\cdot}01h^3$.

$$\int_{5\times 2^{\frac{2}{3}}}^{10\times 50^{\frac{1}{3}}}\int_5^{0{\cdot}01h^3}\frac{de}{e^3}\frac{dh}{h^{\frac{5}{2}}} = -\tfrac{1}{2}\int_{5\times 2^{\frac{2}{3}}}^{10\times 50^{\frac{1}{3}}}\frac{1}{h^{\frac{5}{2}}}\left[\frac{10^4}{h^6} - 2^{\frac{1}{5}}\right]dh$$

$$= \tfrac{1}{50}\tfrac{2}{3}\left[\frac{1}{10\sqrt{5}} - \frac{1}{100\sqrt{5}}\right] - \frac{10000}{15}\left[\frac{1}{10^5 5^{\frac{5}{2}}} - \frac{1}{10^{10}5^{\frac{5}{2}}}\right]$$

$$= 0{\cdot}00053664 - 0{\cdot}00011929$$

$$= 0{\cdot}00041735$$

$$\int_{10\times 50^{\frac{1}{3}}}^\infty\int_{0{\cdot}0001h^3}^{0{\cdot}01h^3}\frac{de}{e^3}\frac{dh}{h^{\frac{5}{2}}} = \tfrac{1}{2}\,1^{\frac{2}{5}}(10^8 - 10^4)\left[\frac{1}{10\times 50^{\frac{1}{3}}}\right]^{\frac{15}{2}}$$

$$= \frac{10^4(10^4-1)\sqrt{5}}{15\,.\,10^{10}\,.\,125}$$

$$= 0{\cdot}00001193.$$

So the integral for the first range is 0·00042928.

We must divide the range $0{\cdot}01h^3 \leqslant e \leqslant h^3$ into two parts, $5 \leqslant h < 5\times 2^{\frac{2}{3}}$ in which we integrate from $e=5$ to $e=h^3$, and $h \geqslant 5\times 2^{\frac{2}{3}}$ in which we integrate from $e=0{\cdot}01h^3$ to $e=h^3$.

$$\int_5^{5\times 2^{\frac{2}{3}}}\int_5^{h^3}\frac{de}{e^3}\frac{dh}{h^{\frac{5}{2}}} = -\tfrac{1}{2}\int_5^{5\times 2^{\frac{2}{3}}}\frac{1}{h^{\frac{5}{2}}}\left(\frac{1}{h^6}-\tfrac{1}{25}\right)dh$$

$$= \tfrac{1}{50}\ \tfrac{2}{3}\ \frac{1}{5^{\frac{3}{2}}}\left(1-\tfrac{1}{2}\right) - \tfrac{1}{2}\ \tfrac{2}{15}\ \frac{1}{5^{\frac{15}{2}}}\left(1-\frac{1}{2^5}\right)$$

$$= 0{\cdot}0005963 - 0{\cdot}00000037$$

$$= 0{\cdot}0005959\ ;$$

$$\int_{5\times 2^{\frac{2}{3}}}^{\infty}\int_{0{\cdot}01h^3}^{h^3} = \tfrac{1}{2}\ \tfrac{2}{15}(10^4-1)\frac{1}{5^{\frac{15}{2}}}\ \frac{1}{2^5}$$

$$= 0{\cdot}0001193\ ;$$

so the integral for the second range is 0·0007152.

The integrals for the other ranges are of the order 10^{-7}.

So we have for the value of the three terms involving Δ but not 3Δ in (56),

$$0{\cdot}00042928\{\tfrac{15849}{4096}\times 0{\cdot}0025 + \tfrac{13041}{8192}\times 0{\cdot}02\}$$
$$+ 0{\cdot}0007152\{\tfrac{15849}{4096}\times 0{\cdot}09 + \tfrac{13041}{8192}\times 0{\cdot}1525\}$$
$$= 0{\cdot}0005.$$

When the factor before the Δ is 3, we have approximately for the first range, $0{\cdot}001h^3 \leqslant e \leqslant 0{\cdot}1h^3$, which is now divided into $h < 50^{\frac{1}{3}}$ in which we do not integrate, $50^{\frac{1}{3}} \leqslant h < 10\times 5^{\frac{1}{3}}$ in which we integrate from $e=5$ to $e=0{\cdot}1h^3$, and $h \geqslant 10\times 5^{\frac{1}{3}}$ in which we integrate from $e=0{\cdot}001h^3$ to $e=0{\cdot}1h^3$.

$$\int_{50^{\frac{1}{3}}}^{10\times 5^{\frac{1}{3}}}\int_5^{0{\cdot}1h^3}\frac{de}{e^3}\frac{dh}{h^{\frac{5}{2}}} = -\tfrac{1}{2}\int_{50^{\frac{1}{3}}}^{10\times 5^{\frac{1}{3}}}\frac{1}{h^{\frac{5}{2}}}\left[\frac{10^2}{h^6}-\tfrac{1}{25}\right]$$

$$= \tfrac{1}{50}\ \tfrac{2}{3}\left[\frac{1}{5\sqrt{2}}-\frac{1}{50\sqrt{2}}\right] - \tfrac{100}{15}\left[\frac{1}{5^5 2^{\frac{3}{2}}}-\frac{1}{50^5 2^{\frac{5}{2}}}\right]$$

$$= \sqrt{10}\times 0{\cdot}00041735$$

$$= 0{\cdot}00125205.$$

So the integral for the whole of this range is 0·00128781, and that for the second range is 0·0021456. The integrals for the other ranges are of the order 10^{-6}; so we have for the value of the three other terms in (56),

$$0{\cdot}00128781\{\tfrac{3969}{1096}\times 0{\cdot}0025 + \tfrac{1809}{8192}\times 0{\cdot}02\} + 0{\cdot}0021456\{\tfrac{3969}{4096}\times 0{\cdot}09$$
$$+ \tfrac{1809}{8192}\times 0{\cdot}1525\} = 0{\cdot}0001.$$

Finally,

$$[e_1] = \frac{16\pi^2\nu a_1^3}{e_1}\ \frac{T}{T_1}\times{\cdot}0006$$
$$= \frac{\nu a_1^3 T}{10 e_1 T_1}.$$

Let us take $T = 10^{13}$ years, $T_1 = 10^{-2}$ years, $a_1 = 10^{13}$ cm. $= 3^{-1}10^{-5}$ parsecs (the separation of the binary is of course $2a_1$), and $\nu = 10$ stars per cubic parsec. Then $[e_1] = 0{\cdot}037/e_1$, which is 0·37 for $e_1 = 0{\cdot}1$.

When we have a factor 2 before the Δ, the integrals will become 0·00085856 and 0·0014304 respectively, so that we have

$$\frac{\delta a_1}{a_1} = \frac{432\pi^2\nu a_1{}^3 T}{T_1}\{[\tfrac{4}{5}\tfrac{1}{1}\tfrac{1}{2} \times 0{\cdot}0015 + \tfrac{1}{1}\tfrac{6}{0}\tfrac{9}{2}\tfrac{}{4} \times (+0{\cdot}011)] \times {\cdot}00085856 + 0{\cdot}0014304[\tfrac{4}{5}\tfrac{1}{1}\tfrac{1}{2} \times 0{\cdot}015 + \tfrac{1}{1}\tfrac{6}{0}\tfrac{9}{2}\tfrac{}{4} \times (+0{\cdot}0825)]\}$$

$$= \frac{0{\cdot}1\nu a_1{}^3 T}{T_1}.$$

With the values given above for ν, a_1, etc., we have

$$\left.\begin{aligned} \frac{\delta a_1}{a_1} &= {\cdot}037 \\ e_1\delta e_1 &= {\cdot}037 \end{aligned}\right\}. \qquad (65)$$

The separation of a spectroscopic binary is not likely to be as large as 2×10^{13} cm., which is 1·29 astronomical units. Both the above expressions vary directly as the cube of the separation.

This paper was begun with the view of trying to find some indication of the reasons for the observed distribution of eccentricity and separation in binary stars and the correlation between these two quantities. Explanation was sought from stellar encounters which were supposed to be sufficiently numerous for an average effect to be taken without appreciable error. The form of the first approximations shows that such effects disappear when the average is taken; the second approximation gives some terms whose averages do not vanish and others for which averages are zero. Even with the large value taken for the initial separation and the high density of stars, on the assumption that binaries originated in clusters, the value of $\delta a_1/a_1$ is too low to be of cosmogonic interest, though the value of δe_1 is considerably higher than might have been expected. We are not entitled to let the initial value of e_1 be less than of the order 0·1 or we shall be guilty of omitting some quantities of the first order of small quantities in our calculations, which will vitiate our supposition that for $e_1 = 0{\cdot}01$, $\delta e_1 = 3{\cdot}7$. In any case we have probably made too generous an estimate of the separation. Equations (65) might suggest $2 \log a_1 = e_1{}^2$ as a law of correlation; but this is far from being the case and we must regard this result as merely a coincidence due partly to our approximations, which may involve errors up to 10 per cent. in the final results.

The work for this paper was begun by Dr. W. M. Smart, to whom section 2 on the general equations of motion is due. I must thank him and Professor H. F. Baker for much valuable assistance and criticism throughout.

OBSERVATIONS OF INTENSITIES OF SOME LINES IN THE SOLAR SPECTRUM.

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§ 1. The invention of spectrophotometry has made available for our study a fresh source of information concerning the outer layers of the Sun and stars, and it has been shown, chiefly by the work of Professor Milne, that the photometric study of stellar spectra can be made to give us stellar temperatures and surface gravities. The experimental difficulties, however, are still so great that the actual yield in information has not yet reached very large proportions, perhaps less than might have been expected a few years ago. Some of these difficulties are of such a nature that we may hope to surmount them in the near future, but some difficulties are not instrumental and may prove to be permanent obstacles to this method of astrophysical investigation. To the former class belong difficulties connected with photographic and photometric technique; and to the latter, the fact that in late type stars the lines are so closely packed in some parts of the spectrum that relatively few can be disentangled, not on account of the finite resolving power at our disposal, but because the lines have a certain width, so that two or more may overlap. We cannot, as we would like to do, pick out in every case the principal lines of the neutral and ionised atom, for comparison. One or both may be badly distorted by its neighbours, or even lost among them. Nevertheless, a great many lines are free from interference, and can be measured for width or for total intensity.

Except in the case of the very strongest lines in the Sun, lack of resolving power makes it extremely difficult for us to conduct measurements of line width, or even of total intensity, to such a degree of perfection as would give an unambiguous answer to the observational questions at present outstanding.

It is difficult to measure the true central intensities of the lines: it is not yet certain that there are anomalies in multiplet intensities: we do not yet know whether the current method of deducing a number of absorbing atoms from a measurement of line intensity is reliable or not, and we are ignorant as to the true calibration of Rowland's scale of line intensities. These problems are observational, because the theoretical questions could be settled immediately if we were in possession of really accurate data.

Most of the high dispersion work carried out, up to the present date, on the solar spectrum has been done with the use of grating spectrographs. It occurred to the writer that his observations,* conducted with the long focus solar spectrograph at Mount Wilson, appeared to indicate that gratings suffer from certain grave disadvantages when used for photometry, and the

* Woolley, *Ap. J.*, **72**, 256, 1930; **73**, 185, 1931. *Observatory*, **55**, 142, 1932.

present note contains an account of observations carried out with a prism spectrograph, from which better results have been obtained. The prism spectra show deeper centres in the lines and are more satisfactory to work with than grating spectra made with the same resolving power. The success of this test is undoubtedly due to the excellent definition of the particular prism spectrograph used, namely, Mr. Evershed's liquid prism apparatus at Ewhurst, but the comparison indicates the general superiority of very good prisms over gratings when used for photometry. In addition to demonstrating the superiority of the prism over the grating, the present observations show the following results: (1) $H\delta$ has a greater intensity in the Sun than $H\gamma$. This fact did not elude the vigilance of Rowland, who gave $H\delta$ the intensity 40, while $H\gamma$ received 20, but it seems to have escaped recent notice. (2) $H\gamma$ and $H\delta$ have certainly not got zero central intensities. Their central intensities must be about 30 per cent. of the intensity of the continuous background. (3) Anomalous intensities occur in very strong multiplets, in circumstances such that the anomaly cannot possibly be attributed to lack of resolving power or to the Doppler effect. (4) It is possible to pick out differences in apparent central intensity among lines of the same total intensity. These differences are characteristic of the types of absorbing systems involved, *e.g.* lines due to *Fe* atoms on the one hand, and to *CN* molecules on the other, fall into different classes, the molecular lines having high apparent central intensities. It is suggested that this change in apparent central intensity is due to a difference in the level at which the absorption of the line takes place, *e.g.* the *CN* molecules are lower in the solar atmosphere than those *Fe* atoms which take part in the formation of the absorption lines. The formation of lines due to ionised atoms takes place at a lower level than that at which lines due to neutral atoms are formed.

§ 2. *Observational Method*

The plates used were cut in halves. One half was exposed, at Ewhurst, in the spectrograph, but not developed. The two halves were then sent to Cambridge, where the unexposed half was standardised, and the two halves developed together. The spectrograms were then examined with the Cambridge microphotometer.

The standardising equipment is a simple box, in which a direct photograph is taken of the image, formed by a cylindrical lens, of a raster which is uniformly illuminated by light from a mercury arc. The arrangement is identical in principle with that used at Mount Wilson by the writer, but the new apparatus contains no grating or other dispersing agent. Instead, sets of Wratten filters are placed in the optical path, to limit the region of the spectrum used in the standardisation. The simplification makes it considerably easier to secure uniformity of illumination on the plate before the raster is put in position, and shortens the exposures necessary to imprint a satisfactory set of standards on the plate. This is an important point in solar work, since the solar spectrograms are taken with relatively short exposures. It takes ten or twenty minutes to make a suitable exposure

with a standardising apparatus, whether of wedge or of raster type, when a prism or grating is used, while the direct photograph takes less than one minute. Working with ordinary plates in the blue-violet, it is safe to assume that no significant change in the calibration curve of a plate appears within one or two hundred angstroms, so that dispersion is unnecessary in a limited spectral region. The simple box is easy to make and to adjust, and has given good service. A diagram is shown in fig. 1.

Two sets of prism spectrograms have been examined. In the first series, E, the spectra were obtained with the liquid 60° prism and solid 30° reflecting prism used in the Littrow form, and equivalent to three 60° prisms. The effective aperture is $3\frac{1}{4}$ inches, the focal length 16 feet, and the dispersion

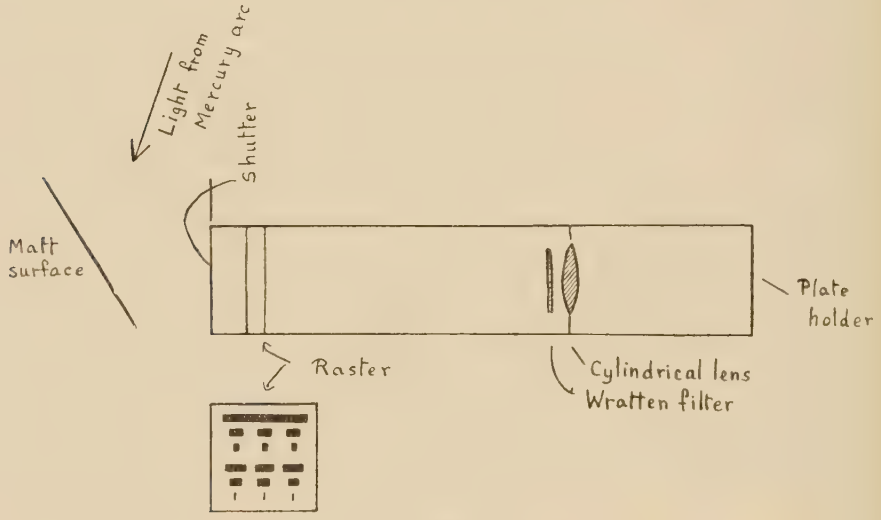

FIG. 1.—*Standardising Arrangements.*

at λ 4200 is 0·64 A. per mm. The theoretical resolving power at λ 4200 is 254,000, and the practical resolving power about 50,000.

In the second series, F, a 60° solid prism was added to the train, increasing the dispersion to 0·38 A. per mm. at λ 4200. The theoretical resolving power was increased, but the practical resolution remained about the same, absorption reducing the effective aperture by an unknown amount.

In addition to examining *Hγ*, *Hδ* and three *Fe* multiplets, all the lines from λ 4086 to λ 4390, whose intensity on Rowland's scale is 1 or greater, were measured, except those so disturbed by their neighbours as to be unmeasurable. In each case the equivalent breadth was determined by plotting the width of the line against r, the ratio of the intensity of light at any point inside the line to the intensity of the continuous background, and measuring the area of the curve $\int_{-\infty}^{\infty}(1-r)d\lambda$ with a planimeter. The measurements of 108 lines are given in Table I. The wave-length, identification and intensity are quoted from the *Revision of Rowland's Table*, and the width, $\lambda-\lambda_0$ at $r=9/10$, and the apparent central intensity, are given

as measured without any correction for finite resolving power. Nearly all the measurements are taken from series E plates, the lines being picked out on account of their freedom from interference by neighbouring lines. A few lines (marked with an asterisk) have been measured on more than one plate, of which at least one is a series F plate, and are included on account of their importance as members of special multiplets. Lines marked with a dagger are difficult to measure on account of interference from their neighbours; an effort has been made to make correct allowances for this interference. Where a line is distorted on one side only, considerations of symmetry help to give the true contour. In the region 4275–4325 A., in which no lines are recorded in Table I, there are so many lines crowded together that few are fit for photometric study, and the region has not been examined in detail.

TABLE I

Measurements of Lines in Interval 4086–4390 *A.*

Wave-length	Element	Rowland Intensity	Width at $r=9/10$. Milli-angstroms	Equivalent Breadth in Milli-angstroms	Central Intensity Per Cent. of Continuous Background
4086·715	*La*$^+$	1	57	28	66
4087·103	*Fe*	3	82	55	49
4088·566	*Fe*	3	70	40	60
4089·226	*Fe*	3	75	40	55
4090·580	*V*	1	75	37	70
4090·962	*Fe*	3	75	42	59
4091·562	*Fe*	3	65	38	58
4094·424	...	2*N*	80	42	69
4094·942	*Ca*	4)	...	60	46
4100·750* †	*Hδ*	40*N*)	2750	1710	27
4107·496	*Fe*	5	135	106	31
4108·536	*Ca*	2	100	60	53
4110·539	*Co*	4	88	75	40
4111·789	*V*	4	103	81	38
4112·325	*Fe*	2	65	36	55
4112·719	*Ti*	1	50	24	67
4114·453	*Fe*	4	94	69	36
4117·858	*Fe*	2	75	40	55
4118·557	*Fe*	5	130	120	30
4120·215	*Fe*	4	75	60	38
4121·327	*Co*	6*d*?	105	96	29
4121·812	Fe, *Cr*	3	75	54	39
4126·523	*Cr*	2	60	32	63
4128·101	*V*	6*d*	90	76	42
4128·744	*Fe*$^+$	2	60	32	64
4129·187	*Cr, Fe*	3	90	60	49
4130·659	*Ba*$^+$	2	60	28	66
4131·121	*Ce*$^+$, *Mn*	1	60	26	68
4132·069*	*Fe*	10	335	320	18
4136·530	*Fe*	4	85	56	48
4137·007	*Fe*	6	120	84	37
4137·659	*Ce*$^+$	1	45	18	74
4139·939	*Fe*	6	80	56	47

Wave-length	Element	Rowland Intensity	Width at $r=9/10$. Milli-angstroms	Equivalent Breadth in Milli-angstroms	Central Intensity Per Cent. of Continuous Background
4140·408	...	3	95	30	56
4141·873	*Fe*	4	67	40	50
4143·880*	*Fe*	15	440	384	20
4145·203	*Fe*	1	45	20	70
4146·990	...	2	57	26	67
4150·972	*Ti, Zr*$^+$	1	70	26	74
4153·391	*Fe*	1	60	26	73
4157·790	*Fe*	5	142	105	34
4158·800	*Fe*	5	100	75	37
4159·188	...	5	125	89	42
4167·279	*Mg*	8	380	267	26
4168·624	*Fe*	2	60	32	62
4168·953	*Fe*	2	57	32	59
4175·645	*Fe*	5	127	98	32
4175·914	...	1	70	31	75
4176·580	*Fe, Mn*	5	150	110	33
4178·863	*Fe*$^+$	3	97	58	45
4180·811	*CN*	2*N*	127	70	56
4187·049* †	*Fe*	6	255	201	30
4187·787* †	*Fe*	5	317	246	30
4187·863	*Fe*	3			
4202·042*	*Fe*	8	480	359	25
4203·573	*Fe, Cr*	2	95	46	62
4203·944	*Fe*	3	125	98	40
4204·006	**Fe**, *La*$^+$	4			
4207·135	*Fe*	3	95	53	55
4207·409	*CN*	1*N*	67	32	68
4208·612	*Fe*	3	88	55	49
4208·987	*Zr*$^+$	1	75	40	66
4210·337	*Fe, Sa*	4	180	140	35
4210·404		3			
4210·970	*CH*	3*N*	87	55	58
4212·644	*Cr*	3*N*	100	59	58
4213·655	*CH*, **Fe**, *CN*	3	117	76	49
4215·545	*CN*, **Sr**$^+$	5*d*?	268	225	28
4216·602	...	1*N*	62	27	70
4217·561	*Fe, La*$^+$, *Cr*	5	120	78	45
4218·726	*CH*	3*N*	83	66	59
4219·357	*Fe*	4	105	88	38
4219·421	*Fe*	3			
4220·053	*V*$^+$	1	62	24	70
4220·349	*Fe*	3	70	40	53
4221·475	...	1*N*	80	54	71
4222·223	*Fe*	5	153	114	38
4224·862	*CH*	2*N*	87	54	56
4226·742	*Ca*	20*d*	1140	912	19
4227·442	*Fe*	4	...	37	29
4231·028	*Ni, CH*	4*N*	90	41	55
4231·956	*Sc*	1	60	25	72
4233·613	*Fe*	6	207	172	32
4235·951	*Fe*	8	345	294	27

Wave-length	Element	Rowland Intensity	Width at $r=9/10$. Milli-angstroms	Equivalent Breadth in Milli-angstroms	Central Intensity Per Cent. of Continuous Background
4238·031	**Fe**, *Sc*	3	160	76	43
4238·818	*Fe*	5	210	160	38
4239·368	...	2	90	48	60
4241·122	*Fe*	2	65	30	70
4243·821	*Fe*	2	65	30	67
4246·094	*Fe*	2	105	76	48
4246·838	Sc^+	5	140	116	34
4250·132* †	*Fe*	8	290	226	27
4250·799* †	*Fe*	8	320	279	22
4254·348	*Cr*	8	250	241	24
4257·663	*Mn*	2	65	38	60
4260·488* †	*Fe*	10	540	470	17
4271·166* †	*Fe*	6	425	328	24
4271·776* †	*Fe*	15	670	555	20
4274·808	*Cr*	7*d*	255	208	27
4325·777 †	*Fe*	8	530	470	25
4331·654	*Ni*	2	65	35	56
4340·477* †	*Hγ*	20*N*	2200	1560	34
4369·781	**Fe**, *CH*	4	160	123	38
4375·946	*Fe*	6	165	127	30
4376·784	*Fe*	1	65	32	60
4378·257	*CH*	2*Nd*?	90	49	56
4379·240	*V*	4	117	94	39
4380·069	*Co*	2*Nd*	75	46	60
4383·559	*Fe*	15	1085	924	16
4387·901	*Fe*	2	70	38	53
4388·416	*Fe*	3	118	70	47
4389·256	*Fe*	2	68	34	54

§ 3. *Comparison of Grating and Prism Spectra*

Table II shows the widths at $r=9/10$, equivalent breadths, and central intensities of thirteen lines observed in the second order grating spectrum

TABLE II

Wave-length	Element	Rowland Intensity	Width at $r=9/10$. Milli-angstroms		Equivalent Breadth in Milli-angstroms		Central Intensity	
4110·539	*Co*	4	80	*90*	57	*72*	50	*42*
4111·789	*V*	4	80	*112*	64	*84*	44	*40*
4112·325	*Fe*	2	70	*65*	36	*36*	60	*55*
4112·719	*Ti*	1	42	*50*	20	*24*	67	*67*
4114·453	*Fe*	4	83	*97*	49	*68*	47	*38*
4117·858	*Fe*	2	57	*75*	32	*40*	61	*55*
4118·557	*Fe*	5	110	*130*	82	*120*	37	*30*
4120·215	*Fe*	4	60	*75*	40	*60*	46	*38*
4246·094	*Fe*	2	110	*105*	75	*76*	54	*48*
4246·838	Sc^+	5	122	*140*	98	*116*	39	*34*
4250·132	*Fe*	8	207	*290*	173	*226*	30	*27*
4250·799	*Fe*	8	275	*300*	228	*264*	30	*25*
4254·348	*Cr*	8	240	*250*	210	*241*	29	*22*

in the 75-foot spectrograph at Mount Wilson, in which the theoretical resolving power is 155,000. The dispersion is 0·35 A. per mm. The corresponding figures in italics are values of the same quantities from Table I, that is to say, taken with a prism spectrograph in which the dispersion is 0·64 A. per mm. at λ 4200. The prism spectra show deeper central intensities and greater total intensities, line for line, than the grating spectra.

In addition to the lines given in Table II, *Hδ* has a lower central intensity (27 per cent. of the continuous background) in the prism spectrograms than in those made with the grating (38 per cent.).

If the differences in central intensity could be attributed to difference in practical resolving power of the two instruments, they would be smaller for wide lines such as *Hδ* and the three lines of Rowland intensity 8 than for the weaker lines. Further, the equivalent breadth is not affected by resolving power. It is clear that some other agency is at work. It is likely that both instruments scatter light which finds its way into the centres of the lines. If we attribute the differences to this cause, the observations show that the grating instrument scatters more light than the prism. It is possible that empirical corrections can be devised for grating spectrographs which would enable one to deduce the true central intensity of a line from the apparent central intensity, but it is certain that the mathematical correction for finite resolving power will not give it. In the writer's view, the prism spectrograph emerges from the present test as the superior instrument for spectrophotometry.

§ 4. *Quality of the Prism Spectrogram. Observed Central Intensities*

The theoretical resolving power of the spectrograph is 254,000 at λ 4200, but the practical resolving power for very narrow solar lines is about one-fifth of this. The central intensities given by the instrument for lines less than 100 milli-angstroms in equivalent breadth depend almost entirely on the equivalent breadths, and are not the true central intensities at all, as the region in which the central intensity is attained is so narrow that the instrument fails to resolve it. The apparent central intensity will be too high for the following further reasons: that a certain amount of light is scattered in the prisms and reflected at lens surfaces (though Mr. Evershed has reduced the latter source of error in his spectrograph); and that a certain amount of light is scattered in the emulsion on the photographic plate.

To return to the question of finite resolving power. This question has been discussed, in the case of gratings, by Carroll * and by Unsöld.† Let us suppose that in the case of a strictly monochromatic line of wave-length λ, the contour produced by the prism is an error curve

$$I = \frac{1}{\beta\sqrt{\pi}} \exp\left\{-\left(\frac{\Delta\lambda}{\beta}\right)^2\right\}, \qquad (4\cdot1)$$

* *M.N.*, **88**, 154, 1927.
† *Ap. J.*, **75**, 106, 1932.

where $\beta = 0{\cdot}6 \times \lambda/R$, and R is the practical resolving power of the prism. Suppose, further, that the true contour of the absorption line is

$$r = 1 - a \exp\left\{-\left(\frac{\Delta\lambda}{\alpha}\right)^2\right\}. \tag{4·2}$$

Then the central intensity recorded by the prism is

$$r_0 = 1 - \frac{a}{\sqrt{1 + (\beta/\alpha)^2}}. \tag{4·3}$$

The equivalent breadth, W, of the line is given by

$$W = a \,.\, \alpha\sqrt{\pi}. \tag{4·4}$$

If we suppose β known, we can determine a and α from the observed values of W and r_0. Against this procedure, it must be noted that the true contour of the line may not be (4.2), and that we may be misled by the presence of scattered light.

There is, however, considerable theoretical ground for supposing that a does not vary greatly from line to line, or at least that there is no systematic variation in a as we go from wide to narrow lines. At any rate, it is interesting to see whether the change in apparent central intensity with equivalent breadth is consistent with the supposition that a is the same for all the lines due to the same element.

Adopting a trial value for a, we calculate α for a given W from (4.4). We then recalculate a from the apparent central intensity r_0, using (4.3).

Fig. 2 shows the apparent central intensities of 32 weak *Fe* lines, from plates in series E, plotted against their equivalent breadths W, and Table III shows values taken from a curve representing mean values. The figure shows that any individual apparent central intensity may differ from the mean value by 5 per cent. or more.

Table III

Apparent Central Intensities of Weak Iron Lines

W in milli-angstroms	20	30	40	60	80
r_0 per cent. of continuous background	75	63	55	45	38

Since the stronger *Fe* lines have apparent central intensities lower than 20 per cent., only values of a equal to or greater than 0·8 can be considered reasonable for the purpose of examining whether the same value of a can fit observations of r_0 for wide and narrow *Fe* lines.

If $R = 254{,}000 \times 1/5$, $\beta = 50$ milli-angstroms. With this value of β, no value of a, subject to the condition, $0{\cdot}8 \geqslant a \geqslant 1$, can be found which will fit the observations of r_0 shown in Table III. We might suppose that the instrument does not perform quite so well as this, and put $\beta = 40$, but the position is not much improved unless we take $a = 0{\cdot}75$, which fits the observations very closely. The true central intensity is then 25 per cent. of the intensity of the continuous background.

An alternative plan is to attempt to make some allowance for scattered light, mentioned earlier in the present section. Suppose that the prism (and emulsion) scatters an approximately uniform amount of light into any frequency interval from neighbouring frequencies. A close fit to the observed values of r_0 can be made by supposing that the instrument scatters 10 per cent. of the background intensity into the middle of a line. The value of a is then 0·9, so that the true central intensity is about 10 per cent. β is again taken as 40. (In this connection it should be mentioned that

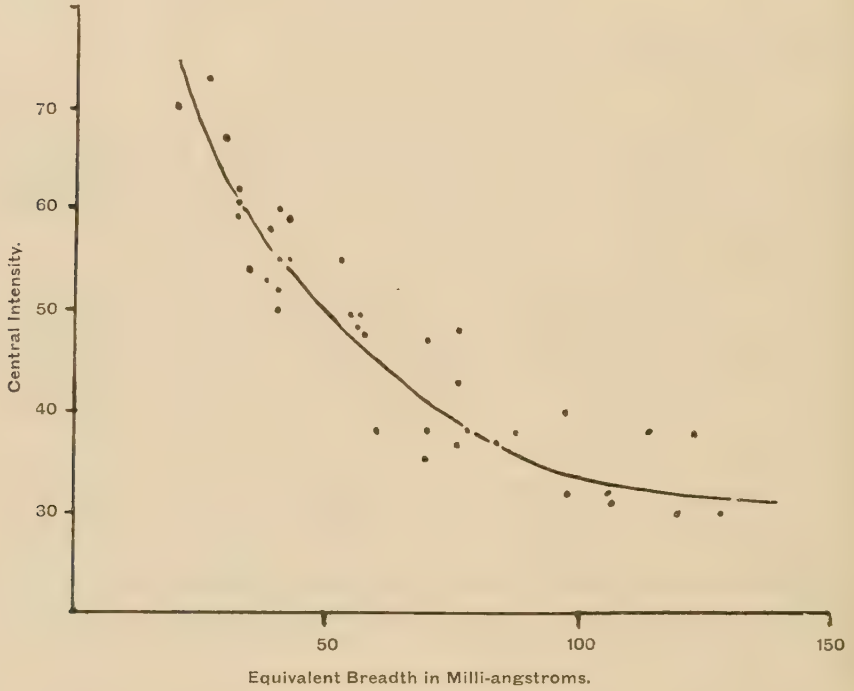

FIG. 2.—*Central intensities of weak* Fe *lines, expressed as percentage of intensity of continuous background, plotted against equivalent breadth in milli-angstroms.*

since the prism spectrograph gives slightly curved lines, it is difficult to ensure that the small part of the line under examination is accurately parallel to the microphotometer slit, and there may be a slight loss of resolving power on this account.)

Although this investigation is necessarily of an approximate nature, we may conclude that the true central intensities of the weak *Fe* lines are probably between 10 and 20 per cent. of the intensity of the continuous background, although the apparent central intensities are as great as 70 per cent. in the case of the weakest lines.

§ 5. *Measurement of* Hγ *and* Hδ

Table IV shows the measurements of *Hγ* and *Hδ*. The lines have been measured in both series E and series F plates.

TABLE IV

Measurement of Hγ *and* Hδ

Line	Rowland Intensity	Width in Angstroms $\lambda - \lambda_0$				Central Intensity	Equivalent Breadth in Angstroms	Approximate Extent of Wings on Each Side of Centre
		$r = \frac{9}{10}$	$\frac{8}{10}$	$\frac{3}{4}$	$\frac{5}{10}$			
Hγ	20	2·20	1·35	1·04	0·29	0·34	1·56	6 A.
Hδ	40	2·75	1·53	1·20	0·31	0·27	1·71	8 A.

The lower central intensity of *H*δ is due, in part, to the *Fe* line, 4101·865, Rowland intensity 3, which depresses the centre. The high central intensities exhibited by these lines are undoubtedly real, since there are many lines which show far smaller apparent central intensities. The figures shown in Table IV for central intensity need diminishing by an unknown amount, owing to finite resolving power of the spectrograph and to scattering of light in the spectrograph and in the emulsion on the plate; but this correction cannot exceed 0·10, which is the lowest central intensity recorded (for λ 4046), since *H*γ and *H*δ have the greatest breadth, and therefore the smallest correction for finite resolving power, of all the lines observed.

The true central intensities of the lines *H*γ and *H*δ, then, are of the order of magnitude of 25 to 30 per cent. of the intensity of the continuous background.

§ 6. *Intensities in Strong* Fe *Multiplets*

Table V shows observations of intensities in three multiplets. In the column "log *N*, observed" is recorded the logarithm of a number proportional to the square of the equivalent breadth. The next column gives the theoretical intensity as given by the "sum rule" of Burger and Dorgelo * and extended to complex multiplets by Russell.† A correction can be applied on the supposition that the atoms in the lower states forming the lines are distributed among these states in the ratios valid for thermodynamical equilibrium at 6000°, the scattering coefficient remaining unchanged. The values of log *N*, so corrected, are also tabulated. An example shows how this correction has been applied. The line λ 4046 is absorbed by atoms in the state a^3F_4, 1·478 volts, and λ 4063 by atoms in the state a^3F_3, 1·551 volts. The weights of these states are 9 and 7 respectively, so that in thermodynamical equilibrium at 6000° the ratio of the number of atoms in the a^3F_4 state to the number in the a^3F_3 state, n_4/n_3, is equal to

$$\frac{9}{7}\left\{\exp\left(-\frac{1\cdot478}{RT}\right)\Big/\exp\left(-\frac{1\cdot551}{RT}\right)\right\},$$

where $RT = 0\cdot518$; therefore $n_4/n_3 = 9/7 \times 1\cdot15$.

Hence the sum rule transition probability is increased by the factor 1·15 for λ 4046 relative to λ 4063.

Most of the lines have been measured repeatedly on both series E and

* Burger and Dorgelo, *Zs. f. Phys.*, **23**, 258, 1924.

† Russell, *Proc. Nat. Acad. Sci.*, **11**, 314, 1925.

series F plates. Three determinations were made of the equivalent breadth of λ 4046, giving the result, $W = 0{\cdot}899 \pm 0{\cdot}024$ A. The most likely source of error is the presence of blends. It is very unusual for a wide line in this part of the spectrum to be free from blends. An attempt has been made to make correct allowances for the distortions of the contours of the lines by blends. In the last column of Table V is given an indication of the state of the line, ranging from *g*, a line free from blends : *fg*, a line in which it is reasonably certain that the correct contour can be drawn without difficulty ; down to *p*, where there is room for some uncertainty. The principal blends are tabulated, the number in brackets being the Rowland intensity of the blending line.

TABLE V

Observed Intensities in strong Fe *Multiplets*

Wave-length	Row-land In-tensity	Type of Multi-plet and Inner Quan-tum Number	Equi-valent Breadth in Ang-stroms	log *N* Observed Solar (present paper)	log *N* Observed Lab. (Frerichs)	log *N* Theoretical Sum Rule	log *N* Theoretical Cor-rected for 6000°	Blends
		$a^3F - y^3F'$						
4045·827	30	4 – 4	·899	0·89	0·91	1·18	1·28	*fg* 4044 (3), 4045 (2), 4045 (5), 4046 (2)
4063·607	20	3 – 3	·699	0·68	0·68	1·02	1·06	*f* 4062 (5), 4063 (4), 4064 (2)
4071·751	15	2 – 2	·568	0·49	0·57	0·90	0·90	*f* 4070 (3), 4070 (4), 4072 (2)
4132·069	10	2 – 3	·322	0·00	0·00	0·00	0·00	*fg* 4131 (2)
4143·880	15	3 – 4	·394	0·09	0·17	0·01	0·05	*fg* 4143 (2)
		$a^3F - z^3G'$						
4147·677	4	4 – 3	·078	0·00	...	0·00	0·00	*fg* 4146 (2), 4147 (2), 4147 (1)
4202·042	8	4 – 4	·359	1·36	...	1·80	1·80	*fg* 4201 (1), 4202 (2)
4250·799	8	3 – 3	·279	1·11	...	1·74	1·74	*f* 4250·132 (8)
4271·776	15	4 – 5	·555	1·71	...	3·09	3·09	*f* 4271·166 (6)
4325·777	8	2 – 3	·544	1·69	...	2·75	2·75	*p* 4325 (4), 4326 (2)
		$z^7D' - e^7D$						
4187·049	6	3 – 2	·200	0·49	0·51	0·53	0·53	*fp* 4186 (2N), 4187 (2)
4187·845	8	4 – 3	·246	0·68	0·52	0·55	0·57	*f* 4187 (2)
4210·370	7	1 – 1	·140	0·18	0·07	0·05	0·02	*fg* 4209 (1), 4211 (3N)
4222·223	5	3 – 3	·114	0·00	0·00	0·00	0·00	*g* ...
4233·613	6	1 – 2	·172	0·43	0·35	0·36	0·33	*fg* 4233 (4)
4235·951	8	4 – 4	·294	0·90	0·60	0·62	0·64	*fg* 4236 (1), 4236 (1)
4250·132	8	2 – 3	·226	0·67	0·50	0·53	0·52	*f* 4250·799 (8)
4260·488	10	5 – 5	·470	1·31	0·97	1·00	1·04	*fp* 4260 (2), 4260 (3)
4271·166	6	3 – 4	·328	1·00	0·49	0·55	0·55	*f* 4271·799 (15)

Without unnecessary repetition, on the present occasion, of recent theory and controversy on this subject, it may be said that on the current theory of line intensities, as developed mainly by Unsöld, we would expect the values of log *N*, as found from the squares of the equivalent breadths, to be equal to the values given by the sum rule. Anomalous multiplet intensities have

been reported before, by Minnaert and his collaborators,* and also by the present writer.† It has, however, been suggested by Minnaert ‡ himself, and also by Unsöld,§ that the anomaly, which consists of the strengthening of the weak members of the multiplet relative to the stronger lines, can be accounted for by taking into consideration the Doppler effect and the effect of finite resolving power of the instrument used.

According to Frerichs,‖ the multiplet $a^3F - y^3F'$ exhibits anomalous intensities in the laboratory. Frerich's measurements for this multiplet, and for $z^7D' - e^7D$, are shown in the column "log N, observed, lab." ($a^3F - z^3G'$ was not measured by Frerichs.) The striking agreement, in the case of $a^3F - y^3F'$, between solar and laboratory values is presumably accidental, since the laboratory intensities are subject to the same temperature correction, when compared with solar intensities, as are the sum-rule intensities, namely, $\Delta \log N = 0{\cdot}10$ for λ 4046. Laboratory measurements agree with sum-rule intensities in normal multiplets, which means that atoms in the arc are distributed among the stationary states in ratios corresponding to infinite temperature. There remains, therefore, an outstanding anomaly between solar and corrected laboratory intensities. The lines given in Table V are so strong that the observations are free from objection on the grounds of Doppler effect and finite resolving power. Note that there is no *systematic* anomaly in the case of $z^7D' - e^7D$.

§ 7. *Calibration of Rowland's Scale of Intensities*

The arithmetic means of the equivalent breadths of the lines in Table I, grouped together according to their Rowland numbers, are shown in Table VI. Mean values of the equivalent breadths of lines whose Rowland numbers exceed 6 are not given, since the Rowland intensities do not define the equivalent breadth of a line at all closely when the line is of strength 8 or greater.

The values found by Minnaert in the region λ 5200 are shown, for comparison.¶ They are much larger, for a given Rowland number, than the values here given for λ 4200. This means that there is a colour effect in the Rowland scale; the intensity 4 in the yellow indicates twice as much equivalent breadth as it does in the violet. This is a known effect, which may be seen from Minnaert's comparison of his own results at λ 5200 and λ 4500 respectively.**

The calibration of the Rowland scale given in Table VI is in all respects similar to results found previously,†† and presents no fresh features.

* Minnaert and van Assenbergh, *Zs. f. Phys.*, **53**, 248, 1929; Minnaert and Mulders, *Zs. f. Astrophys.*, **1**, 192, 1930.

† Woolley, *Ap. J.*, **72**, 256, 1930.

‡ Minnaert and Mulders, *Zs. f. Astrophys.*, **2**, 165, 1931.

§ Unsöld, *Ap. J.*, **75**, 106, 1932.

‖ *Ann. d. Phys.*, **81**, 807, 1926.

¶ Minnaert and Mulders, *Zs. f. Astrophys.*, **1**, 192, 1930.

** *Ibid.*

†† Minnaert, *ibid.*; Woolley, *Ap. J.*, **72**, 361, 1930.

Table VI

Calibration of Rowland's Scale of Intensities

Rowland Number		1	2	3	4	5	6
Equivalent breadth in milli-angstroms	(1) Present paper	28	38	52	70	102	108
	(2) Minnaert, λ 5200	57	84	117	161	237	203

§ 8. We have already seen that the instrumental distortion of a very narrow line is so great that it is difficult to apply corrections and arrive at the true contour. Nevertheless, it is possible to use the observations shown in Table I to demonstrate that, with a given equivalent breadth, differences in contour do exist. We can actually classify lines according to their type of contour, and it appears that lines due to *Fe* have smaller apparent central intensities than lines, with the same equivalent breadth, due to *CH* and *CN*. This difference has been recognised by spectroscopists working before the introduction of spectrophotometry, and Rowland's use of the letter N to qualify the intensity of a line is found to coincide with the spectrophotometric description that the line has a high apparent central intensity.

The effect can be exhibited from the prism spectrograms in the following way. The apparent central intensities of 76 lines, having equivalent breadths of less than 100 milli-angstroms, were plotted against the equivalent breadths, and a smooth curve was drawn through the points so obtained. This curve shows the mean apparent central intensity for each equivalent breadth.

The ratio of the actual apparent central intensity of each line to the mean apparent central intensity for lines of that equivalent breadth was found. The mean values of this ratio for various classes of lines are shown in Table VII.

Table VII

Apparent Central Intensity Ratio

No. of Lines	Nature of Lines	Mean Ratio
31	*Fe*	·93
9	Other neutral metals	·92
8	Ionised metals	·98
4	*V*	1·03
6	*CH* and *CN*	1·10
4	Listed by Rowland as having the characteristic N	1·18

The other 15 lines are blends, etc., and are not classified in Table VII.

The cause of this variation in contour is undoubtedly variation in vertical distribution, in the Sun's atmosphere, of the systems concerned in the formation of the lines. From a complete knowledge of the contour of the line it should be possible to calculate the actual distribution of the element concerned, but this could hardly be done at present. We could, however, work the other way, and, assuming certain distributions in the solar atmosphere, calculate the true contours, and also the apparent central intensities, seen in an instrument of finite resolving power. Short of calculating the

actual distribution of *Fe* and of *CN* in the solar atmosphere and predicting contours, we can give a simple illustration of the way in which the depth at which an absorbing layer occurs modifies the contour of the resulting line. The fact that an absorbing layer, limited in extent, creates a line with less total absorption if it is at a low level in the Sun than if it is at a high level, has been pointed out by Eddington. We can make use of the same analysis to show further that there is a corresponding difference in contour, the lines formed deep down having a higher central intensity. We use Eddington's results* for a thin but opaque layer, at an optical depth τ below the surface. Then

$$1 - r = \frac{3\eta\delta\tau}{e^{z_1} + \sqrt{3}\sinh z_1 \,.\, \eta\delta\tau},$$

where $\eta = l_\nu/k$, the ratio of the coefficient of line absorption to the coefficient of continuous absorption.

$\delta\tau$ = the optical thickness of the layer.

$z_1 = 1{\cdot}32 + \sqrt{3} \,.\, \tau$.

Table VIII shows values of r, the blackening produced by an absorbing layer at various depths.

TABLE VIII

Change in Contour with Depth of Absorbing Layer

	τ, Depth of Layer		
Value of $\eta\delta\tau$	0	$\frac{1}{3}$	1
0·1	0·93	0·96	0·986
0·2	·86	·92	·975
1	·56	·76	·92
2	·39	·67	·89
10	·09	·53	·84
∞	·003	·47	·73

These contours are shown in fig. 3. The contour of the line produced by a layer at a finite depth has the character of the contours observed in the case of the *CN* molecular lines. There is nothing surprising in this, as we should expect the molecules to lie at a certain depth, since at lower pressures they would dissociate. But it may seem surprising to find that the lines due to ionised metals have higher apparent central intensities than those due to neutral metals, which appears to indicate that the layers concerned in the formation of the enhanced lines are lower than those in the case of the neutral lines. However, increase of temperature, as well as decrease of pressure, brings about the dissociation of an atom into ion and electron. The practical part of an absorption line (that is to say, all except the centre) is formed effectively at a fairly low optical depth, and it appears that this depth occurs below the point where increase of temperature with increasing optical depth more than compensates for the increase of pressure, so that below this point, where the dissociation is a minimum, the dissociation actually increases with optical depth. The calculation of the dissociation of

* Eddington, *M.N.*, **89**, 627, 1929.

Ca in the Sun has been given by the writer (*M.N.*, 92, 496, 1932, Table IV). From the figures there given it can be seen that the dissociation of *Ca* has a minimum near $\tau = \frac{1}{6}$. Since the optical depth which acts as a sort of centre of gravity of the formation of the line is $\tau = \frac{1}{3}$, where the dissociation is

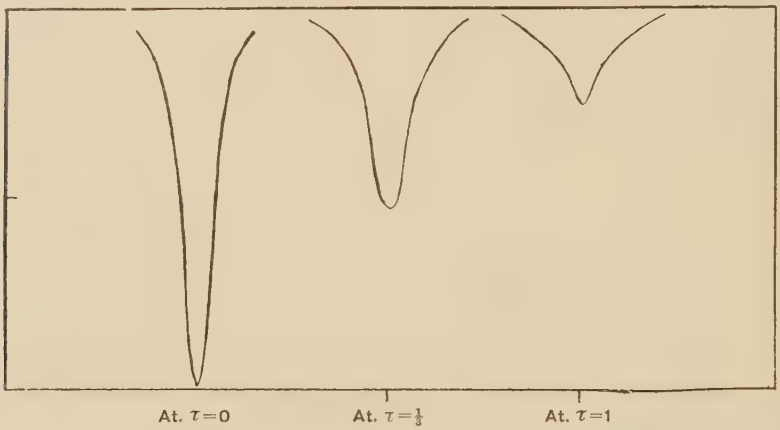

FIG. 3.—*Theoretical contours illustrating differences in the shape of a line formed at different depths. The absorbing layer shown for* $\tau = 1$ *contains three times as many atoms as the layer shown for* $\tau = 0$.

increasing with τ, it becomes intelligible that the enhanced lines have slightly higher apparent centres than the neutral lines. The writer hopes to give a detailed justification of this point on another occasion.

The writer wishes to thank Mr. J. Evershed very warmly. Mr. Evershed not only consented to the use of his spectrograph, but went further and did the work of exposing the plates himself.

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THE HYDROGEN EMISSION IN GASEOUS NEBULÆ.

G. Cillié.

(Communicated by E. A. Milne)

Scope of the Paper.—According to Zanstra's theory of nebular luminosity the hydrogen spectrum in nebulæ is emitted on the recombination of protons and electrons. Expressions for the numbers of captures on the various hydrogen levels are here derived. They are used to predict the Balmer emission in nebulæ.

In addition the free-free emission for a mixture of protons and electrons is calculated. This determines the strength of the continuous nebular emission.

1. *Introduction.*—In a gaseous nebula, photo-electric ionisation of hydrogen is the principal mechanism by which energy from the associated star is absorbed.* Owing to the dilution of radiation, ionisations occur from the ground level only. The nebula therefore absorbs radiation of wave-lengths beyond the Lyman limit, while allowing radiation of greater wave-lengths to pass through freely.

The effective temperatures of the stars associated with planetary nebulæ are of the order 40,000°.† We assume here a Maxwellian velocity-distribution for the free electrons corresponding to somewhat lower temperatures. According to Eddington the photo-electrons in the cosmic cloud take up a temperature of the same order as that of the exciting stars.‡ There, however, "nothing important" happens to the electrons between ejection and re-capture. In nebulæ, on the other hand, they lose a considerable amount of their kinetic energy in exciting nebulium emission.§

Gaunt has given an expression for the probability of the photo-electric ionisation of a hydrogen atom in the nth state.‖ By this, a value for the probability of capture of an electron in this state is derived from Milne's relation ¶ between these two probabilities. We use this value to calculate the number of electrons captured per second on the various hydrogen levels for given conditions of density and temperature.

A quantum, belonging to the continuous spectrum at the head of the nth series, is emitted for every capture on the nth level. Subsequently the electron cascades down to the ground level, with the emission of one or more line quanta. While in an excited state the atom is not affected by absorptions or collisions, owing to the dilution of the radiation and the low density. Using the numbers of electrons caught on the various levels, we calculate, by means of the Einstein coefficients, the relative intensities of the Balmer line emissions. Since these coefficients are known only as far as the seventh,** captures on higher levels are neglected.

Carroll's discussion of the physical state of hydrogen in nebulæ †† was based on Plaskett's observations of the Balmer decrement.‡‡ In the present method this decrement is predicted theoretically.

By a method due to Milne §§ a relation between the probabilities of absorption and emission for free-free electron switches is derived. This, together with Gaunt's value for the probability of absorption,‖‖ leads to an expression for the continuous emission of a mixture of protons and electrons.

* Zanstra, *Pub. D.A.O.*, **4**, 200, 1931, and *Zs. f. Astrophysik*, **2**, 1, 1931. The present paper develops some of the ideas expressed in these papers concerning the physical nature of nebulæ.

† Zanstra, *Zs. f. Astrophysik*, **2**, 329, 1931, Table I.

‡ Eddington, *Proc. Roy. Soc.*, A, **111**, 424, 1926.

§ Bowen, *Ap. J.*, **67**, 1, 1928.

‖ Gaunt, *Phil. Trans.*, 229 A, 200, 1929.

¶ Milne, *Phil. Mag.*, **47**, 209, 1924.

** Slack, *Phys. Rev.*, **31**, 527, 1928.

†† Carroll, *M.N.*, **90**, 588, 1930.

‡‡ H. H. Plaskett, *H.C.O. Circular*, 335, 1928.

§§ Milne, *loc. cit.*, §§ 8 and 9.

‖‖ Gaunt, *loc. cit.*

This result is used to calculate the energy of the general emission in a gaseous nebula. In comparison with this the general absorption is negligible, owing to the weakness of the radiation field.

2. *The Number of Captures on the Various Hydrogen Levels.*—Let n_1, n_2 and y be the number of hydrogen atoms, protons and free electrons per c.c. respectively in an enclosure. If the temperature is T, the number of electrons per c.c. having velocities in the range v to $v+dv$ is

$$4\pi y\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{1}{2}mv^2/kT}v^2dv, \tag{1}$$

where m is the mass of the electron and k Boltzmann's constant. We define $F_n(v)$, the probability that a proton captures a v-electron on the nth level, in such a way that the number of captures of v-electrons on the nth level per second per c.c. is

$$n_2y8\pi^2\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} e^{-\frac{1}{2}mv^2/kT}F_n(v)v^3dv. \tag{2}$$

For each such capture a quantum, $h\nu$, is emitted, whose energy is given by

$$h\nu=\chi_n+\tfrac{1}{2}mv^2, \tag{3}$$

where χ_n is the nth ionisation potential of hydrogen.

Let $\psi_n(\nu)I(\nu)d\nu$ be the probability that an atom in the nth state absorbs in one second a quantum of energy $h\nu>\chi_n$, with a frequency in a range $d\nu$, when exposed to an isotropic radiation field, $I(\nu)d\nu$. Milne has shown * that

$$\psi_n(\nu)=\frac{(2\pi mc)^2}{(h\nu)^3}\frac{q}{q_n}F_n(v)v^2, \tag{4}$$

where q and q_n are the statistical weights of the proton and the hydrogen atom in the nth state respectively, and where v and ν are connected by (3).

Apart from a factor of the order unity, Gaunt gives the following value for $a_1(\nu, n)$,† the rate of absorption of energy, by a hydrogen-like atom in the nth state, when exposed to isotropic ν-radiation of unit intensity, viz.

$$a_1(\nu, n)=\frac{2^8\pi^5}{3\sqrt{3}}\frac{mZ^4\epsilon^{10}}{ch^6\nu^3n^5}, \tag{5}$$

where $Z\epsilon$ is the effective nuclear charge. From their definitions, $a_1(\nu, n)=h\nu\psi_n(\nu)$, so that by (2), (4) and (5) the number of captures of v-electrons per c.c. per second on the nth level is

$$n_2y\frac{2^9\pi^5}{(6\pi)^{\frac{3}{2}}}\frac{\epsilon^{10}}{mc^3h^3}\left(\frac{m}{kT}\right)^{\frac{3}{2}}\frac{1}{n^3}\frac{e^{-\frac{1}{2}mv^2/kT}}{\chi_n+\frac{1}{2}mv^2}vdv, \tag{6}$$

where we have put $Z=1$, $q=1$ and $q_n=n^2$ for hydrogen.

Gaunt showed that for small n, (5) is a good approximation near the series limit (v small). According to subsequent investigations ‡ it holds

* Milne, *loc. cit.*, equation (17).

† Gaunt, *loc. cit.*

‡ Strömgren, *Zs. f. Astrophysik*, 4, 118, 1932.

over wide ranges of velocities for $n = 1$ and 2. It certainly breaks down for very high velocities. We are justified, however, in integrating (6) over all velocities on account of the exponential factor. Hence by (6) and (3) we find the total number of captures per second per c.c. on the nth level,

$$n_2 y \frac{2^9\pi^5}{(6\pi)^{\frac{3}{2}}} \frac{\epsilon^{10}}{m^2c^3h^3}\left(\frac{m}{kT}\right)^{\frac{3}{2}} \frac{1}{n^3} e^{\chi_n/kT} Ei(\chi_n/kT), \tag{7}$$

where $Ei(x_0) = \int_{x_0}^{\infty} e^{-x}dx/x$. The number of captures on the nth level is proportional to

$$M(n, T) = \frac{1}{T^{\frac{3}{2}}} \frac{1}{n^3} e^{\chi_n/kT} Ei(\chi_n/kT). \tag{7'}$$

In Table I we give values for $M(n, T)$ for a range of temperatures. The table illustrates two physically obvious facts. First, given two values of n_2 and y, the rate of recombination on any level decreases with increasing temperature—faster electrons are more evasive than slow ones. Secondly, the decrement in $M(n, T)$ becomes more steep as the temperature increases —faster electrons tend to be caught relatively more often on the lower levels.

TABLE I

Values of $M(n, T) \times 10^8$

n	$T = 1000°$	$T = 5000°$	$T = 10{,}000°$	$T = 20{,}000°$	$T = 50{,}000°$
1	20	8·8	6·0	3·9	2·3
2	9·8	3·9	2·7	1·6	0·78
3	6·4	2·5	1·4	0·86	0·37
4	4·7	1·6	0·94	0·52	0·21
5	3·5	1·1	0·64	0·34	0·13
6	2·9	0·86	0·46	0·23	0·088
7	2·3	0·66	0·35	0·17	0·062
8	1·9	0·52	0·26	0·13	0·046
9	1·6	0·41	0·21	0·099	0·034
10	1·4	0·34	0·17	0·078	0·027

3. *The Hydrogen Line Emission in Nebulæ.*—Let N_n be the number of hydrogen atoms per c.c. in the nth state at any moment. Let $A_{s,r}$ be the transition probability between the two states s and r $(s > r)$. The total number spontaneously leaving the nth state per second per c.c. is $\sum_{r=1}^{n-1} N_n A_{n,r}$. The total number arriving in this state per second per c.c. from higher states is $\sum_{r=1}^{\infty} N_{n+r} A_{n+r,n}$. The number arriving by direct capture is, by (7) and (7'), $n_2 y C M(n, T)$, where we write C for the constant in (7).

A nebula is transparent to the subordinate hydrogen emissions. Hence

the above are the only three ways in which electrons can leave or arrive in the nth state, apart from collisions and Lyman line absorption. The effect of collisions is negligible on account of the low density. Lyman line absorption from the stellar radiation is unimportant. The nebula itself emits Lyman line radiation, and with Zanstra we assume that there is complete Lyman reabsorption, no Lyman quanta, other than Lyα, leaving the nebula. For if the number of times that, say, a Lyβ quantum is absorbed before it can escape is big enough, it will be converted into a Lyα and an $H\alpha$ quantum.

In this case the number of spontaneous transitions from the nth to the ground level is on an average equal to the number of arrivals in the nth state by Lyman reabsorption. Assuming the whole to be in equilibrium, we find

$$N_n \sum_{r=2}^{n-1} A_{n,\,r} = n_2 y C M(n,\,T) + \sum_{r=1}^{\infty} N_{n+r} A_{n+r,\,n}. \tag{8}$$

Or, introducing

$$B_n = \sum_{r=2}^{n-1} A_{n,\,r},$$

$$n_2 y C M(n,\,T) = N_n B_n - N_{n+1} A_{n+1,\,n} - N_{n+2} A_{n+2,\,n} \cdots, \tag{8'}$$

which holds for $n = 3,\ 4,\ 5$, etc. From (8) we have similarly,

$$n_2 y C M(n+1,\,T) = N_{n+1} B_{n+1} - N_{n+2} A_{n+2,\,n+1} - N_{n+3} A_{n+3,\,n+1} \cdots \tag{8''}$$

and

$$n_2 y C M(n+2,\,T) = N_{n+2} B_{n+2} - N_{n+3} A_{n+3,\,n+2} - N_{n+4} A_{n+4,\,n+2} \cdots \tag{8'''}$$

Eliminating N_{n+1} between (8') and (8'') we find

$$[B_{n+1} M(n,\,T) + A_{n+1,\,n} M(n+1,\,T)] n_2 y C = N_n B_n B_{n+1} - N_{n+2} [A_{n+2,\,n} B_{n+1} + A_{n+2,\,n+1} A_{n+1,\,n}] \cdots \tag{8''''}$$

We next eliminate N_{n+2} between (8''') and (8''''), and proceed in this way to find better approximations to N_n. With these solutions the Balmer decrement is found from

$$\frac{I(\nu_{n2})}{I(\nu_{m2})} = \frac{N_n A_{n,\,2} \nu_{n2}}{N_m A_{m,\,2} \nu_{m2}}, \tag{9}$$

where I denotes energy.

Neglecting atoms in states above the seventh and using Slack's values of $A_{r,\,s}$, we thus find from (8),

$$10^8 \frac{N_3}{n_2 y C} = 2{\cdot}262 M(3,\,T) + 1{\cdot}169 M(4,\,T) + 1{\cdot}095 M(5,\,T) + 1{\cdot}054 M(6,\,T) + 1{\cdot}035 M(7,\,T), \tag{α}$$

$$10^8 \frac{N_4}{n_2 y C} = 5{\cdot}719 M(4,\,T) + 2{\cdot}087 M(5,\,T) + 1{\cdot}808 M(6,\,T) + 1{\cdot}860 M(7,\,T), \tag{β}$$

$$10^8 \frac{N_5}{n_2 y C} = 13{\cdot}37 M(5,\,T) + 3{\cdot}700 M(6,\,T) + 3{\cdot}240 M(7,\,T). \tag{γ}$$

Using Table I we find the values of $N_n/n_2 y C$ given in Table II. From Table I we know that the number of electrons caught on the nth level per

TABLE II

Values of $\frac{N_n}{n_2 \gamma C}$

n	$T=10{,}000°$	$T=20{,}000°$	$T=50{,}000°$
3	0·059	0·033	0·0139
4	0·083	0·044	0·0175
5	0·114	0·060	0·0229

second decreases as n increases. Table II shows that the number in the nth state at any moment increases with n. This is due to the increase with increasing n of the average mean life of an atom in an excited state.

TABLE III

Balmer Decrement

	$T=10{,}000°$	$T=20{,}000°$	$T=50{,}000°$
H_α	27·5	29·1	30·8
H_β	10·0	10·0	10·0
H_γ	4·64	4·57	4·40

In Table III we give the theoretical Balmer decrement derived from Table II. H_β is taken arbitrarily as 10 in each case.

4. *Captures on Levels higher than the Seventh.*—In deducing the results of the previous section we neglected emissions resulting from captures of electrons on levels above the seventh. We derive an upper limit for these. In Table I we evaluated $M(n, T)$ up to $n=10$; hence this estimate involves, by (7), finding an upper limit for

$$S=\sum_{n=11}^{\infty}\frac{1}{n^3}e^{\chi_n/kT}Ei(\chi_n/kT).$$

Since $\chi_n=\chi_1/n^2$ we may write this as

$$S=\sum_{n=11}^{\infty}\frac{1}{n^3}e^{A/n^2}Ei(A/n^2), \qquad (10)$$

where $A=\chi_1/kT$. Since A/n^2 is small for the range of T we are considering, we may write

$$Ei(A/n^2)=-\gamma-\log_e A/n^2+A/n^2-\frac{1}{2\cdot 2!}(A/n^2)^2+\cdots$$

$$<-(\gamma+\log_e A)+2\log_e n+A/n^2,$$

where $\gamma = 0{\cdot}5772$ (Euler's constant). Hence

$$S < 2e^{A/121} \sum_{n=11}^{\infty} \frac{1}{n^3} \log_e n + A \sum_{n=11}^{\infty} \frac{1}{n^5} e^{A/n^2} - (\gamma + \log_e A) \sum_{n=11}^{\infty} \frac{1}{n^3} e^{A/n^2}. \quad (11)$$

If $f(r)$ is a uniformly decreasing function of r,*

$$\sum_{r=s}^{\infty} f(r) > \int_s^{\infty} f(x)dx > \sum_{r=s+1}^{\infty} f(r),$$

where r takes integral values only. Applying this theorem to (11), we find

$$S < \tfrac{1}{200} e^{A/121}\{2 \log_e 10 + 1\} + \tfrac{1}{200}\{e^{A/100} - 1\} - \frac{1}{2A}(\gamma + \log_e A)\{e^{A/121} - 1\}. \quad (12)$$

For $T = 10{,}000°$, $A = 15{\cdot}68$, and we find $S < 0{\cdot}01701$. Hence, by Table I,

$$\sum_{n=8}^{\infty} M(n,\ 10{,}000°) < 2{\cdot}34 \times 10^{-8}. \quad (13)$$

Hence for $T = 10{,}000°$ the captures neglected are somewhat less than those on the second level.

All these electrons cascade down to the ground level, giving rise directly or indirectly to Balmer emissions. They therefore affect our numerical values given in Table II. On the other hand, since they are captured mainly on very high levels, the third, fourth and fifth levels are not affected very differently by them, so that our values for the Balmer decrement in Table III will not be changed considerably by these captures.

5. *Comparison with Observation.*—Table IV reproduces the relative intensities of the Balmer emissions in planetary nebulæ as measured by

TABLE IV

The Observed Balmer Decrement

Line	Plaskett								Berman				
N.G.C.	1976	2440	3242	I.C. 3568	I.C. 4593	6790	7662	7027	7027	6543	6826	6572	7009
H_α	40	...	...	...	...	...	...	58	...	...	...	...	...
H_β	10	10	10	10	10	10	10	10	10	10	10	10	10
H_γ	4·8	3·5	4·1	4·8	5·6	3·9	3·9	3·4	3·9	6·7	5·7	5·8	5·9

Plaskett and Berman.† The theoretical values of H_α/H_β and H_γ/H_β found in § 3, corresponds, as regards order of magnitude, with observation. The

* I gratefully acknowledge the help of Mr. W. L. Ferrar at this stage.

† Plaskett, *loc. cit.*, and Berman, *L.O.B.*, 430, 1930.

theory fails to explain the observed dispersion in H_γ/H_β. The failure may be due to our assumption, inherent in the application of Slack's numerical values for the coefficients $A_{r,s}$, that at any moment the substates of any level are equally occupied.

6. *The Total Energy Emitted per Second by a Typical Planetary Nebula in its Balmer Line Spectrum.*—If we assume complete Lyman reabsorption (see § 3), all captures on the third and higher levels give rise to Balmer line emissions. Let M_N be the mass of the nebula, a fraction α by weight being hydrogen, of which a fraction β is ionised. Then, assuming that the nebula is spherical (r_1) and, further, that the density is uniform, we find

$$n_2 = \frac{3M_N\alpha\beta}{4\pi r_1^3\mu_H},$$

where μ_H is the atomic weight of hydrogen. Hydrogen appears to be very abundant in nebulæ. We know, further, that the oxygen in planetary nebulæ is mostly doubly ionised. Hence the ionisation of the hydrogen will be practically complete. So that we have, approximately,

$$y = n_2 = \frac{3M_N}{4\pi r_1^3\mu_H}.$$

Hence by (6) the number of Balmer line quanta emitted per second by the nebula is

$$\frac{3M_N^2}{4\pi r_1^3\mu_H^2}\,\frac{2^9\pi^5}{(6\pi)^{\frac{3}{2}}}\,\frac{\epsilon^{10}}{m^2c^3h^3}\left(\frac{m}{kT}\right)^{\frac{3}{2}}\sum_{n=3}^{\infty}\frac{1}{n^3}e^{\chi_n/kT}Ei(\chi_n/kT). \qquad (14)$$

Adopting the current estimates, $M_N = 10^{31}$ grm., $r_1 = 10^{17}$ cm. and $T = 10{,}000°$, we find that the nebula emits about $1{\cdot}8 \times 10^{45}$ Balmer quanta per second. A mean wave-length for these of 5000 A. reduces this to an energy of about 7×10^{33} ergs per second, which is of the observed order of magnitude. The calculation constitutes a rough order of magnitude check on the adopted dimensions of a typical nebula. According to these there are about 10^3 hydrogen atoms per c.c., practically all of which are ionised at any moment. Every electron is captured once in about 3×10^9 seconds. This is the duration of its "mean free life."

7. *The Continuous Balmer Emission in Nebulæ.*—The continuous Balmer emission results from the captures on the second level. By (6) the energy emitted per second per c.c. in a range $d\nu$ of this spectrum is

$$n_2 y\,\frac{2^9\pi^5}{(6\pi)^{\frac{3}{2}}}\,\frac{\epsilon^{10}}{m^2c^3h^2}\left(\frac{m}{kT}\right)^{\frac{3}{2}}\tfrac{1}{8}e^{(\chi_2-h\nu)/kT}d\nu, \qquad (15)$$

where $h\nu > \chi_2$. The spectrum starts from a maximum intensity at the series limit, and fades out according to the law $e^{(\chi_2-h\nu)/kT}$. This law is sensitive to changes in T. It therefore seems possible that a careful study of the intensity distribution in the continuous Balmer spectrum will yield information about the electron temperatures in nebulæ.

Integrating (15) from $\nu = \chi_2/h$ to ∞, we find that the total energy emitted per c.c. per second in this spectrum is

$$n_2 y \frac{2^6\pi^5}{(6\pi)^{\frac{3}{2}}} \frac{\epsilon^{10}}{mc^3h^3}\left(\frac{m}{kT}\right)^{\frac{1}{2}}. \tag{16}$$

This is of the same order as the H_α emission for $T = 10{,}000°$.

8. *The General Emission in Gaseous Nebulæ.*—When an electron makes a free-free absorption of a quantum, $h\nu$, it gains kinetic energy according to the equation

$$\tfrac{1}{2}m(v^2 - v'^2) = h\nu, \tag{17}$$

where v' and v are its velocities before and after absorption respectively. The converse of this process occurs when an electron makes a spontaneous transition from one hyperbolic orbit to one of less energy with the emission of a quantum. The low densities in nebulæ tend to slow down both these processes considerably. The dilute radiation field still further slows down the direct process as compared with conditions of thermodynamic equilibrium, but has no effect on its converse. Hence, in nebulæ, free-free absorption is negligible compared with free-free emission.

Consider an enclosure containing n_2 ions of effective nuclear charge $Z\epsilon$ and y free electrons per c.c. Let the number of electrons which have velocities in a range dv be $y(v)dv$. Any of these electrons, which passes within distances p and $p+dp$ from an ion in the short time dt, we call a (v, p) electron. Let $f(v, p, \nu)dv$ be the probability that in one second a (v, p) electron makes a free-free transition, with the emission of a quantum whose energy lies in the range $h\nu$ to $h(\nu+d\nu)$. The number of transitions per second per c.c. of v-electrons emitting quanta in the above range is

$$n_2 y(v)vdvd\nu\int_0^\infty 2\pi pf(v, p, \nu)dp \equiv 2\pi n_2 F(\nu, v)y(v)vdvd\nu. \tag{18}$$

In (18) we have neglected so-called stimulated transitions. In the case of bound-free transitions Milne has shown that, in order to obtain consistent results, it is necessary to take these into account. The same is true of the present case. On the introduction of a term for them, (18) becomes

$$2\pi n_2[F(\nu, v) + I(\nu)G(\nu, v)]y(v)vdvd\nu, \tag{18'}$$

where $I(\nu)$ is the intensity of ν-radiation, assumed isotropic. In (17) we regard v as fixed, so that apart from sign,

$$mv'dv' = hd\nu. \tag{17'}$$

Hence we may write (18') also as

$$2\pi n_2 m/h[F(\nu, v) + I(\nu)G(\nu, v)]y(v)vdvv'dv'. \tag{18''}$$

Consider now the converse process. An electron, of velocity v', absorbs a quantum whose energy is in the range $h\nu$ to $h(\nu+\delta\nu)$, and its final velocity lies in the range v to $v+\delta v$. Hence, by (17),

$$mv\delta v = h\delta\nu. \tag{17''}$$

Let $\phi(v', \nu)I(\nu)\delta\nu\delta t$ be the probability that this happens in time δt, where $I(\nu)$ measures the intensity of isotropic ν-radiation. The number of such absorptions per c.c. per second is

$$n_2 y(v')\delta v'\phi(v', \nu)I(\nu)\delta\nu,$$

which, by (17″), may be written

$$n_2 m/h\phi(v', \nu)I(\nu)y(v')\delta v' v\delta v. \tag{19}$$

Suppose now that conditions of thermodynamic equilibrium obtain in the enclosure; then if the temperature is T, $y(v)dv$ is given by (1) and

$$I(\nu) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}. \tag{20}$$

Further, if in (18″) and (19) we identify dv and δv, dv' and $\delta v'$, we find, by the principle of detailed balancing,

$$2\pi[F(\nu, v) + I(\nu)G(\nu, v)]y(v)v' = \phi(v', \nu)I(\nu)y(v'), \tag{21}$$

where v, v' and ν are connected by (17).

Substituting in (21) for $y(v)$ and $y(v')$ from (1), and solving for $I(\nu)$, we find on using (17) that

$$I(\nu) = \frac{2\pi F(\nu, v)v^2}{\phi(v', \nu)e^{h\nu/kT}v' - 2\pi G(\nu, v)v^2}. \tag{22}$$

Now (20) and (22) should be identical, hence *

$$F(\nu, v)/G(\nu, v) = 2h\nu^3/c^2 \tag{23}$$

and

$$F(\nu, v)v^2 = h/\pi c^2\phi(v', \nu)v'\nu^3. \tag{24}$$

According to Gaunt † the rate of absorption of ν-energy by an ion with charge $Z\epsilon$, per unit density of v'-electrons, per unit intensity of ν-radiation, is

$$a_2(v', \nu) = \frac{16\pi^2}{3\sqrt{3}}\frac{Z^2\epsilon^6}{hcm^2}\frac{1}{v'\nu^3}. \tag{25}$$

From their definitions, $a_2(v', \nu) = h\nu\phi(v', \nu)$, so that

$$\phi(v', \nu) = \frac{16\pi^2}{3\sqrt{3}}\frac{Z^2\epsilon^6}{h^2cm^2}\frac{1}{v'\nu^4}, \tag{26}$$

and, by (24),

$$F(\nu, v)v^2 = \frac{16\pi}{3\sqrt{3}}\frac{Z^2\epsilon^6}{hc^3m^2} \cdot 1/\nu. \tag{27}$$

By means of this explicit value for $F(\nu, v)$ it is possible to calculate the free-free emission of a mixture of protons and electrons. The effect of the stimulated transitions is proportional to the radiation field. As we are applying our formulæ to nebulæ where this is very weak, we neglect the

* Compare Milne, *loc. cit.*, equations (16) and (17).

† Gaunt, *loc. cit.*

stimulated effect. From (1), (18) and (27) we find that the number of spontaneous transitions of v-electrons, which give rise to emissions of ν-radiation, is

$$n_2 y \frac{2^7\pi^3}{(6\pi)^{\frac{3}{2}}}\left(\frac{m}{kT}\right)^{\frac{3}{2}}\frac{\epsilon^6}{hc^3m^2}e^{-\frac{1}{2}mv^2/kT}v dv\frac{d\nu}{\nu}, \tag{28}$$

where we put $Z=1$ for hydrogen. To find the total number of transitions per c.c. per second, which give rise to the emissions of $h\nu$-quanta, we integrate (28) with respect to v between the limits given by $\frac{1}{2}mv^2=h\nu$ and $v=\infty$. This gives

$$n_2 y \frac{2^7\pi^3}{(6\pi)^{\frac{3}{2}}}\left(\frac{m}{kT}\right)^{\frac{1}{2}}\frac{\epsilon^6}{hc^3m^2}e^{-h\nu/kT}\frac{d\nu}{\nu}. \tag{29}$$

The energy emitted in this range, $d\nu$, is

$$n_2 y \frac{2^7\pi^3}{(6\pi)^{\frac{3}{2}}}\left(\frac{m}{kT}\right)^{\frac{1}{2}}\frac{\epsilon^6}{c^3m^2}e^{-h\nu/kT}d\nu. \tag{30}$$

This gives the energy distribution in the continuous emission of the mixture. It rises to a maximum at low frequencies. The total energy emitted in this spectrum per c.c. per second is

$$n_2 y \frac{2^7\pi^3}{(6\pi)^{\frac{3}{2}}}\left(\frac{kT}{m}\right)^{\frac{1}{2}}\frac{\epsilon^6}{hc^3m}. \tag{31}$$

In actual nebulæ very little of this energy is reabsorbed. We may therefore use (31) directly to estimate the general emission. To avoid the uncertainties concerning dimensions, we compare this energy with that emitted in the Balmer continuous spectrum, (16). The ratio is

$$\frac{2}{\pi^2}\frac{h^2k}{m\epsilon^4}T. \tag{32}$$

In particular, for $T=10{,}000°$ this expression is 0·25. This shows why in actual nebulæ this emission is never detected ; for, although its total energy is of the same order as that in the Balmer continuous spectrum, it is distributed over a wider range of frequencies and contained in the infra-red region.

9. *Conclusion.*—Without distinguishing between the sub-states of hydrogen, we find no marked change in the theoretical Balmer decrement with electron temperature (Table III). The observed value of $H\gamma/H_\beta$ varies considerably from nebula to nebula (Table IV). Hence, assuming that this variation is a temperature effect, we have that although the temperature has little influence on the relative numbers of captures on the different levels (Table I), it greatly alters their distribution amongst the sub-states.

With current values for the dimensions of a typical planetary nebula we calculate approximately the total energy emitted in its Balmer line spectrum. This provides an order of magnitude check on the assumed dimensions.

The energy-distribution in the Balmer continuous spectrum is found to be very sensitive to changes in the electron temperature. It is suggested

that observations of this spectrum be used to determine the temperature directly.

In addition to this recombination spectrum, a nebula has a general emission owing to the high state of ionisation of the hydrogen. It is shown that this emission is too faint to be observed.

In conclusion, it is a pleasure to record my thanks for helpful discussion and criticism to Professor E. A. Milne and Dr. J. Woltjer, jun.

Hertford College, Oxford :
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NOTE ON MILNE'S THEORY OF STELLAR STRUCTURE.

Henry Norris Russell.

1. Professor Milne's important paper * does much to dispel the misunderstandings which have arisen regarding the theory of stellar structure. He does well to insist on the value of rational and rigorous solutions of simplified forms of the very complex problem—solutions which, though they may not be good approximations to the actual configurations, may give very valuable guidance as to the possible *types* of configurations.

His conclusion that configurations in which the material is everywhere perfectly gaseous, and which have masses and luminosities comparable with those of actual stars, must have radii of the order of magnitude of a parsec, is, however, so remarkable as to merit careful examination. This conclusion depends upon a very elegant transformation of the differential equations, which brings to light two new characteristic lengths, associated with the problem, and leads to new equations free from numerical coefficients ; and also upon a conjecture regarding the solutions of the latter equations.

Should the solution actually be made, all uncertainty would disappear. Meanwhile a different line of approach may illustrate the physical meaning of Milne's characteristic lengths, and of his conjectures.

These lengths are defined by the equations,

$$l' = \left(\frac{c\sqrt{3}}{L}\right)^{\frac{1}{2}} \frac{(R/\mu)^2}{(4\pi)^{\frac{1}{2}} G(\frac{1}{3}a)^{\frac{1}{2}}}, \qquad \text{(M26)}\dagger$$

$$\lambda_1{}'^{m+3n} = \frac{aL}{4\pi cGM} \frac{(R/\mu)^{m+2n}}{(4\pi G)^{\frac{1}{2}m+\frac{3}{2}n}(\frac{1}{3}a)^{\frac{1}{2}(m+n)}}, \qquad \text{(M15)}$$

where the opacity coefficient is

$$\kappa = aT^m\rho^n \qquad \text{(M9)}$$

and the other symbols have their usual meanings.

* *Zeitschrift für Astrophysik*, 4, 75, 1932.

† These are Milne's numbers for the equations.

Milne introduces also dimensionless "discriminants" defined by the equations,

$$C = \frac{(R/\mu)^4}{4\pi(\frac{1}{3}a)G^3M^2}, \tag{M18'}$$

$$C' = (\lambda_1'/l')^2. \tag{M25}$$

Physical interpretations of these quantities may be derived from their algebraical forms, independently of the way in which these were derived. The density and total pressure at any point inside the body may be defined by *

$$\rho = X\bar{\rho}, \qquad P = Y\bar{P}, \tag{1}$$

where

$$\bar{\rho} = \frac{3}{4\pi}\frac{M}{r_1^3}, \qquad \bar{P} = \frac{3G}{8\pi}\frac{M^2}{r_1^4}, \tag{2}$$

and X and Y are functions of r, whose values depend on the solution of the differential equations.

Eddington's equation may then be written

$$\frac{1-\beta}{\beta^4} = \frac{\pi}{18}\frac{aG^3}{(R/\mu)^4}\frac{Y^3}{X^4}M^2 = \frac{W^3}{24C}, \tag{3}$$

if we set

$$W = Y/X^{\frac{4}{3}}. \tag{4}$$

At the centre W cannot exceed unity, unless the central density is less than that in some outer region. For the maximum central pressure is obtained by bringing all the mass as far in as possible—under the limitation, into a homogeneous core,—in which case $Y_c = X_c^{\frac{4}{3}}$. In the outer regions we may have $W > 1$, for example, in polytropes with $n > 3$.

When $C = 1$, radiation-pressure is therefore just beginning to become significant in the interior of the mass.

2. The temperature at any point within the body is given by

$$T = \frac{G}{R/\mu}\frac{M}{2r_1}\frac{\beta Y}{X}, \tag{5}$$

and the boundary temperature (according to Schwarzschild, Hopf and Bronstein) by

$$[T(r_1)]^4 = \frac{\sqrt{3}}{ac}\frac{L}{4\pi r_1^2}. \tag{M5}$$

From (M26) we find

$$T(r_1) = \frac{R/\mu}{(\frac{4}{3}\pi G a r_1 l')^{\frac{1}{2}}} = \frac{GM}{R/\mu}\left(\frac{C}{r_1 l'}\right)^{\frac{1}{2}}, \tag{6}$$

whence

$$T/T(r_1) = \frac{1}{2C^{\frac{1}{2}}}\left(\frac{l'}{r_1}\right)^{\frac{1}{2}}\beta W X^{\frac{1}{3}} = \frac{3^{\frac{1}{3}}}{C^{\frac{1}{6}}}\left(\frac{1-\beta}{\beta}\right)^{\frac{1}{3}}X^{\frac{1}{3}}\left(\frac{l'}{r_1}\right)^{\frac{1}{2}}. \tag{7}$$

This equation is true for all gaseous bodies. It does not involve the law of energy-generation at all, and the law of opacity comes in only in determining

* *Astronomy* (Russell, Dugan and Stewart), p. 896.

the relation between the boundary temperature and the effective temperature. It is still valid in the gaseous parts of a body with a degenerate—or even a solid—core, since only the gravitational attraction of this part has to be considered in deriving (3).

Now T is necessarily greater than $T(r_1)$. For a small mass, C is large and $1-\beta$ small. If $W<1$ and r_1/l' is of the order unity, X must be very great. In this case Milne's conjecture is tantamount to assuming that the central condensation, even of a nearly isothermal mass, is great—and of a mass with a hot centre very much greater. For large masses C and β are small. There must be some part of the mass denser than the mean (with $X>1$). If this is still gaseous r_1/l' must be large for a roughly isothermal body (nebula?) and may be so even if the centre is hot.

3. If q is the radiation-pressure at any point,

$$q/P=1-\beta \tag{8}$$

and

$$\frac{dq}{dP}=\frac{\kappa\eta}{4\pi cG}, \tag{9}$$

where $\eta=L(r)/M(r)$.

Taking κ from (M9), using (2) and (4), and then (M18′) and (M15), we find

$$\frac{dq}{dP}=\frac{3^n}{2^m C^{\frac{1}{2}(m+n)}}\frac{\eta M}{L}\left(\frac{\lambda_1'}{r_1}\right)^{m+3n}(\beta W)^m X^{n+\frac{1}{3}m}. \tag{10}$$

Setting $m=-\frac{7}{2}$, $n=1$, this becomes

$$\frac{dq}{dP}=24\sqrt{2}C^{\frac{5}{4}}\frac{\eta M}{L}\left(\frac{r_1}{\lambda_1}\right)^{\frac{1}{2}}(\beta W)^{-\frac{7}{2}}X^{-\frac{1}{6}},$$

whence

$$\frac{r_1}{\lambda_1'}=\frac{1}{1152C^{\frac{5}{2}}}\left(\frac{L}{\eta M}\right)^2(\beta W)^7X^{\frac{1}{3}}\left(\frac{dq}{dP}\right)^2=\frac{3^{\frac{1}{3}}}{C^{\frac{1}{6}}}\left(\frac{1-\beta}{\beta}\right)^{\frac{7}{3}}X^{\frac{1}{3}}\left(\frac{L}{\eta M}\right)^2\left(\frac{dq}{dP}\right)^2. \tag{11}$$

This equation involves the assumed law of opacity and any arbitrary law of generation of energy (through η), but is still true for the gaseous part of a body even if the interior is non-gaseous.

With Milne's assumption of a constant rate of generation, $L/\eta M=1$.

For small masses, q/P is small, and dq/dP will be so too, except under rare conditions. In any case, if $dq/dP>1$ the gas-pressure and density must decrease inward. Otherwise r_1/λ_1' must be small—indeed, very small —unless the central condensation is extreme. For large masses $\frac{dq}{dP}=1$, and, as before, if the whole mass is gaseous $\frac{r_1}{\lambda_1}$ must be large.

4. If the body is to be a star, and not a nebula, the optical thickness from centre to boundary must be great. This is defined by

$$\tau_c=\int_0^{r_1}\kappa\rho dr=\alpha\int_0^{r_1}T^m\rho^{n+1}dr \tag{12}$$

(provided that the mass is wholly gaseous). There is no doubt that m is negative and n positive. If $\rho_{\max}$ is the maximum internal density, we have

$$\tau_c < \alpha[T(r_1)]^m[\rho_{\max}]^{n+1}r_1.$$

The inequality may be very great. Setting $\rho_{\max} = X\bar{\rho}$ we have, for the superior limit,

$$\tau_{\max} = \alpha X^{n+1}[T(r_1)]^m\bar{\rho}^{n+1}r_1. \tag{13}$$

Substituting from (2) and (6) we find

$$\tau^2_{\max} = \frac{X^{2n+2}3^{1+\frac{1}{2}n-\frac{1}{2}m}\alpha^2L^{m+3n+2}}{(4\pi)^{2n+2}M^{4n+2}(R/\mu)^{2m}G^{3n-m+2}c^{m+3n+2}C^{3n+2}}\left(\frac{l'}{r_1}\right)^{m+6n+4}.$$

From (M15) and (M18′) we find

$$\lambda_1'^{2m+6n} = \frac{\alpha^2L^2(cG^2M^2)^{m+n}}{(4\pi G)^{2n+2}(R/\mu)^{2m}c^2M^2},$$

and from (M26) and (M18′),

$$l'^2 = c\sqrt{3}CGM^2/L,$$

whence, by (M25),

$$C'^{m+3n} = \frac{\alpha^2L^{m-3n+2}}{3^{\frac{1}{2}(m+3n)}(4\pi)^{2+2n}M^{4n+2}(R/\mu)^{2m}G^{3n-m+2}c^{m+3n+2}C^{2n}},$$

so that

$$\tau^2_{\max} = \frac{3^{2n+1}C'^{m+3n}X^{2n+2}}{C^{n+2}}\left(\frac{l'}{r_1}\right)^{m+6n+4}. \tag{14}$$

Setting $m = -\frac{7}{2}$, $n = 1$, we have

$$\tau_{\max} = \frac{\sqrt{27}}{C^{\frac{9}{2}}C'^{\frac{1}{4}}}\left(\frac{l'}{r_1}\right)^{\frac{13}{4}}X^2. \tag{15}$$

The conditions $C = C' = 1$, $r_1 = l'$, demand that $\tau'_{\max}$ shall be small—that is, that the body shall be translucent to the centre—a nebula, not a star—unless X is great. Even in this case the wide margin afforded by our assumptions indicates that the outer portions, at least, of the body will be transparent.

5. The results derived above for small masses rest on the assumption that $\frac{d\rho}{dr}$ is negative, which must now be justified. From (9) and (M9) we have

$$dq = \tfrac{4}{3}aT^3dT = \frac{\alpha\eta}{4\pi cG}T^m\rho^n dP, \quad \text{whence, since} \quad \beta P = R\rho T/\mu,$$

$$AT^{3-m+n}dT = \beta^m P^n dP, \quad \text{where} \quad A = \frac{16\pi cGaR^n}{3\alpha\eta\mu^n}.$$

Within the greater part of a small mass we may set $\beta = 1$.

Integrating,

$$P^{n+1}/(n+1) = A(T^{4-m+n} - U)/(4-m+n), \tag{16}$$

when U is a constant of integration.

Then

$$\rho^{n+1}/(n+1) = \mu^{n+1}A(T^{3-m} - UT^{-n-1})/(4-m+n)R^{n+1},$$

whence we find

$$(4-m+n)\frac{R}{\mu}T\frac{d\rho}{dP} = 3-m+(n+1)UT^{m-n-4}. \quad (17)$$

In practice m is negative and n positive, so that $d\rho/dP$ must be positive unless U is negative—in which case, by (16), P is finite when T falls to zero. Such solutions are physically meaningless. Now dP/dr is necessarily negative, hence $d\rho/dr$ is so also in all physically significant solutions.

When $U=0$, the solution goes over into a polytrope with index $n'=(3-m)/(n+1)$, as was first shown by Jeans.* With $m=-\frac{7}{2}$, $n=1$, $n'=3\frac{1}{4}$ and $(1-\beta)/\beta^4 \propto \rho^{-1\frac{1}{3}}$, and the approximation $\beta=1$ fails near the boundary—as Jeans points out.

If the central temperature considerably exceeds that at the boundary, the polytrope $n=3\frac{1}{4}$ will be a good approximation throughout most of the mass.

Milne's analysis therefore is not inconsistent with the existence of gaseous configurations, with ϵ constant, of small mass and radius; nor does the present analysis disprove the existence of gaseous configurations of small mass, very large radius and low temperature throughout.

For large masses, in which β is small, Jeans showed † that the purely gaseous part should arrange itself like a polytropic sphere with $n=7$, which, if of finite mass and radius, must have a non-gaseous core.

For intermediate masses, he gives an approximate solution ‡ showing that the central regions of the body will follow the polytrope $n=3\cdot5$ for a mass 2·5 times the Sun's, 4·0 for 10⊙ and 4·5 for 29⊙ (for a mean molecular weight of 2·5, otherwise varying as μ^{-2}).

This indicates that the transition from "small" masses, which under the given assumptions can exist with small radii and gaseous to the centre, to "large" masses, which cannot, occurs toward the upper end of the observed range of stellar masses.

6. The writer would prefer to state the fundamental problem of stellar structure in somewhat different language from Professor Milne, namely: "What are the possible configurations of aggregates of matter of mass M which can radiate energy steadily at the rate L for an interval of millions of years or longer?" This formulation includes the possibility that very dense bodies, such as the white dwarfs, may maintain their radiation by gravitational contraction for a long time. It is agreed that more diffuse bodies must derive substantially all the energy that they radiate from other less easily exhaustible internal sources. Milne's phrase "endowed with sources totalling to L" might be interpreted to mean "provided with sources of *fixed* total strength L" or "possessing sources which, under the actual internal conditions, total L." He has made it quite clear that the first of these is a misunderstanding.

* *M.N.*, 85, 203, 1925.

† *Loc. cit.*; also *Astronomy and Cosmogony* (Cambridge, 1928), p. 89.

‡ *Astronomy and Cosmogony*, pp. 92–96.

The foregoing statements of the problem become definite only when the physical properties of the material—molecular weight, opacity, etc.—are specified, preferably by means of the atomic composition ; and when some assumption is made regarding uniformity of composition.

If only we knew the law of generation of energy the problem might be restated: "What is the atomic composition of a mass M which, in a steady state, radiates at the rate L?" Since atomic composition involves many disposable constants, this problem will probably in general be indeterminate, even if we add the restriction that the radius shall have the observed value. In the absence of this knowledge, we must make some postulate regarding the law of energy-generation. The simplicity of the assumption, $\epsilon =$ constant, fully justifies a detailed investigation of its consequences.

Princeton University Observatory :
1932 *May* 20.

ON THE N. TEMPERATE BELT OF JUPITER AND ON SYSTEMATIC ERROR IN OBSERVING TRANSITS.

A. Stanley Williams.

The region of the N. temperate belt of Jupiter, in about 25° N. latitude, is one of the most interesting regions of the planet. This is because in nearly the same N. latitude have been found not only the spots having the swiftest known motion, but also those having the slowest motion, with the exception of certain abnormal spots at the south edge of the S. equatorial belt. This interesting peculiarity has now been known for over forty years.* It seems a remarkable fact that the swiftest known motion should occur, not near the equator, as in the case of the Sun, but in about 25° N. latitude.

The present position with regard to the matter has been recently summarised clearly and succinctly by the Rev. T. E. R. Phillips in the following words: "Spots at the S. edge of the belt are not often apparent, or at any rate very conspicuous, and are apt to escape [even] systematic observation, but in 1880, 1891, 1926 (one spot) and 1929 such spots showed a rotation period of about $9^h\ 49^m$, which is considerably shorter even than that of equatorial spots." † Spots and streaks on the N. edge of the N. temperate belt, which is often double, have a rotation period as long as $9^h\ 56^m \pm$. But in 1928 Phillips found for the preceding end of a darker portion of the belt, at the time apparently single, the abnormal intermediate period of $9^h\ 53^m\ 10^s$.

During the past opposition of the planet several dark streaks, or darker portions of the belt, were visible. These showed an abnormal intermediate rotation period similar to that of 1928. Some observations of these by Phillips and B. M. Peek have already been published in the paper just

* For an early reference as to this, see *The Observatory*, **15**, 112, 1892, where attention was called to the peculiarity of the existence of two atmospheric currents close together but differing in velocity by about 280 miles an hour.

† *J.B.A.A.*, **42**, 145, 1932.

cited. These observations relate to the centres of the dark streaks. A few observations of the centres were also obtained here, but most of my observations relate to the preceding ends of the streaks as being more definite and easier to observe than the centres or following ends. For this reason, and because of the systematic error referred to in the latter part of this paper, it has seemed preferable to consider my observations by themselves without reference to those of other observers.

Below are given the longitudes derived from my transit observations of three of the dark N. temperate streaks. The observations were made with a $6\frac{1}{2}$-inch reflector and a power of 150. The weights are on a scale ranging from 1 (bad) to 5 (very good).

Date 1932	Streak α		Streak β		Streak γ	
	Long.	Wt.	Long.	Wt.	Long.	Wt.
	°		°		°	
Jan. 11	...	...	...	...	360·1	1
13	...	...	...	...	357·8	2
24	235·7	2	...	...	...	...
31	...	...	238·2	1	...	...
Feb. 1	190·4	est.	236·9	2	276·2	2
10	151·7	2	205·8	est.	...	...
12	143·9	3	198·3	3	232·1	3
15	...	...	184·9	2	...	...
18	110·6	2	...	...	...	...
22	...	...	158·7	2	...	...
Mar. 16	356·8	1	...	...	...	...
18	343·9	3	...	...	...	...
23	319·3	est.	(20·0)	2	...	...
24	...	...	...	...	69·4	2
26	...	...	21·4	2	60·0	2
30	293·1	3	...	...	...	...
Apr. 8	252·7	3	...	...	...	...
11	...	...	300·3	1	...	...
18	...	...	280·5	1	...	...
20	(213·0)	2	277·6	2	...	...
25	...	...	255·2	2	...	...
May 6	127 ±	...	...	...	...	...
7	...	...	203·4	est.	(239·7)	1

These streaks were quite plain and easy on any fairly good night, though unfortunately the seeing during the spring was very bad indeed. However, there can be no uncertainty with regard to correct identification, which, of course, is of paramount importance in all work of this kind. The preceding end of the streak α had at its extreme s.p. tip a little dark projection. On February 12, a night of unusually good seeing, this projection appeared as a minute, round, black spot on the extreme S. edge of the belt, and even projecting slightly beyond it. The following rotation periods were derived from the observations:—

	Rotation Period			Rotations	Observations
	h	m	s		
Streak α	9	52	33·5	248	12
,, β	9	52	52·3	235	13
,, γ	9	52	53·7	283	7

The mean rotation period is $9^h\ 52^m\ 46^s{\cdot}5$ (3 spots). The motion of α was rather quicker than that of the other two streaks. But it seems remarkable that the individual streaks should have moved at nearly uniform rates. Irregularities of motion might rather have been expected in the case of an abnormal rotation period, nearly midway between the two extremes of $9^h\ 49^m$ and $9^h\ 56^m$, if it were due to the intermingling of the two adjacent currents.

Ever since the red spot became the object of general observation, over fifty years ago, it has been known that observations of the transits of spots are subject to systematic error, as is the case with all observations involving bisection. The matter seems to have been first investigated by Julius Schmidt, of Athens, who found systematic differences in the times of transit ranging from $-5^m{\cdot}83$ to $+3^m{\cdot}60$, as compared with his own.* It should be remarked, however, that many of the twenty different observers, whose results were considered by him, must have been quite inexperienced.

It is not, however, proposed to enter here into the subject of the general systematic error of observation, but merely to call attention to what seems a somewhat remarkable systematic difference in the times of transit when observed with the right and left eye, respectively, of the same observer. I cannot recall exactly when first I noticed that there was a difference between my two eyes, but it must have been a long time ago, since the matter was referred to in a little volume of observations of Jupiter published in 1889.† But from 1886 onwards up to 1928 care was taken to make all my observations of transits with the right eye. However, from and after 1929 practically all my observations have been made with the left eye. This was done partly because perception with the right eye had deteriorated, and partly in order to keep this eye for variable star work, using the left eye for observing a bright object, such as Jupiter.

My attention was redirected to the subject by a remark by Phillips that my observations of 1928 and 1929 seemed to be systematically early compared with those of other observers, and investigation has shown that this is undoubtedly the case. Moreover, the difference is so great that I have been able to detect it directly, by observing rapidly the same spot with both eyes at the same transit. On 7 nights, when a very definite spot has been visible, the transits were observed with the two eyes as under :

	L. Eye. G.M.T.	R. Eye. G.M.T.	L. − R.	Spot
1928	h m	h m	m	
Dec. 7	22 18	22 21·5	−3·5	N. tropical spot.
1929				
Jan. 3	19 12·5	19 16·5	−4·0	S. equatorial spot.
Sept. 30	2 54	2 56·5	−2·5	N. tropical spot.
Oct. 1	4 06·5	4 10	−3·5	,, ,,
5	3 25	3 28·5	−3·5	,, ,,
17	3 38·5	3 42·5	−4·0	,, ,,
18	2 59	3 03	−4·0	,, ,,

* *A.N.*, 2410, 1881.

† *Zenographical Fragments*, **1**, 4, 1889.

The mean difference L. – R. is $-3^{m}\cdot5$, corresponding to $2^{\circ}\cdot1$ of longitude. It follows from this that the longitudes in the first part of the present paper, and in fact in all my work since 1928, except where otherwise expressly mentioned, should be increased by 2° (or more exactly $2^{\circ}\cdot1$) in order to make them conform to my right-eye observations, or to those of other observers in general.

I believe that my right-eye observations have always been in close agreement with those of other observers, and that there is no reason to suspect any alteration. Up to the present time I have not been able to find a satisfactory explanation of the cause of this systematic difference. A black spot central on a white disc always remains central when viewed by either eye. But the effect of a black spot in the centre of a *luminous* disc has not been tried yet. Systematic differences are not generally of much consequence except when comparing or combining the observations of different observers, when, if known, they can be allowed for.

Polvarth, St. Mawes :
1932 *May* 30.

A SIMPLE STELLAR PHOTOMETER.

W. H. Steavenson.

The most successful visual photometers have been those in which the star to be observed and some standard object are brought to equality by an absorbing, occulting or polarising device. In the case of the Harvard Meridian Photometer the standard object was a real star—Polaris. But this is not practicable when observations are to be made with an equatorial, and an artificial star, produced in the instrument, is generally employed. The artificial star is formed at the side of the photometer, and its image is brought by some reflecting arrangement into the optical axis of the telescope.

Among the objections to this type of photometer are the difficulty of making the artificial standard look like a real star and the loss of light due to the interposition of the reflecting device in the cone of rays from the objective. This second drawback is more particularly operative in instruments where a half-silvered cube is used, which makes it impossible to apply the photometer to very faint stars.

I have recently devised and brought into use a photometer of a very simple type, in which these defects, though still existent, are considerably reduced. A plate of polished glass, about 0·75 inch square and 0·1 inch thick, is placed in the focus of the objective or mirror of the telescope, its plane being normal to the optical axis. In the surface furthest from the objective a series of minute dents has been made with a finely pointed diamond. These dents, which are nearly circular and about 0·002 inch in diameter, are illuminated through the thickness of the glass by a small electric bulb at the side. Owing to the irregular form of the depressions the light is reflected towards the eye-piece by only a relatively small portion of each, and the

resulting appearance is that of a point-like object very similar to the image of a real star as seen with the telescope.

The field of view is otherwise dark, since the light from the side is totally reflected by the two polished surfaces of the glass plate, and so cannot illuminate any fragments of dust which may lie upon them. As a further precaution the sides and edges of the plate have been coated with black varnish.

The eye-piece used is of the so-called monocentric type, being a triple-cemented combination with only two air-glass surfaces. Thus only four air-glass surfaces are interposed between the objective and the eye, and this, combined with the darkness of the field, makes it possible to observe and measure the brightness of objects as faint as could normally be seen with any of the ordinary two-lens types of eye-piece.

The source of light is a 3·5-volt flash-lamp bulb of the "daylight" type, fed by a 3-volt source, transformed down from the main supply at 220 volts A.C. Suitable colour-screens can be inserted to suit the spectral type of the star to be observed. There is an achromatic condensing lens for the light, and the brightness of the artificial star is controlled by a sliding wedge placed between the condensing lens and the glass plate. The wedge most generally used is about 3 inches in length. Its gradation is shallow, 1·81 inches corresponding to an absorption of one magnitude. For the measurement of ranges greater than 1·5 magnitudes there is an alternative wedge of the same length but with a steeper gradation, amounting to 0·53 inch per magnitude.

Actually there is no scale engraved on either wedge, each observation being recorded by making a mark with a sharply pointed pencil on a piece of card held against the fitting in which the wedge slides. A separate piece of card is used for each measure or pair of measures, and the positions and separation of the marks can be read off and reduced later. This obviates the necessity of using a light to take the readings, so that the eye can be kept in a sensitive condition for the observation of very faint stars.

This "glass-plate photometer," as I have termed it, acts in a purely differential way. The star to be measured is brought close to one side of the artificial star in the field of view. The light is then adjusted, either by a rheostat or by altering the focus of the condenser, until the artificial star appears of about the same brightness as the real one. If conditions are good, the two will then appear very much alike ; but on all but the steadiest nights the real star will appear somewhat enlarged. If such enlargement is not excessive, it can be quite closely matched in the artificial star by a slight derangement of the focus of the eye-piece. The wedge is now manipulated until both objects appear of sensibly equal brightness, and the card is marked. When this is done, the whole telescope is turned to some star in the North Polar Sequence which is not very different in magnitude from the object just observed. Apparent equality is again brought about by moving the wedge, and the card is marked as before.

A sliding device, worked from outside by a screw, allows of a selection being made among the artificial stars, which appear of very different magnitudes according to the angle at which the light strikes them. As the cone of

radiation entering the eye-piece from each one is far from being of uniform intensity, it is important that the emergent pencil of rays should be kept smaller than the pupil of the eye by means of a diaphragm. Also, where the power used is a relatively high one, such diaphragm should be about equal in diameter to the Ramsden disc. Otherwise the spurious images of the real and artificial stars will appear of different sizes, and thus give rise to errors in estimating their relative brightness.

The instrument works very satisfactorily and is quite easy to use. Its first trial was made on a pair of stars differing by about half a magnitude. Ten pairs of settings were made, and the greatest deviation of any one of them from the mean of the ten was $0^m.07$, and the average deviation, $0^m.048$. Further practice ought to ensure even more consistent results, especially under good conditions.

The actual construction of the photometer was entirely the work of my friend, Mr. H. J. Quilter, who spent a great deal of time over the difficult business of making the minute dents in the glass. The calibration of the wedges was kindly carried out for me at the Royal Observatory by Mr. Greaves and Mr. Martin, who used a microphotometer for the purpose.

OBSERVATIONS OF COMETS MADE WITH THE DORIDIS REFRACTOR (GAUTIER, 400 MM.) OF THE NATIONAL OBSERVATORY OF ATHENS.

S. Plakidis.

(Communicated by A. C. D. Crommelin)

Date	G.C.T.	$\Delta\alpha$	$\Delta\delta$	Number of Comparisons	$\alpha_{1932\cdot0}$	log Parallax Factor	$\delta_{1932\cdot0}$	log Parallax Factor	*
			Comet Houghton-Ensor (1932 *b*)						
1932	h m s	m s	′ ″		h m s		° ′ ″		
Apr. 26	21 30 44	+1 51·58	+5 2·9	12 : 12	12 44 7·23	8·946	−24 4 49·9	0·887	1
,, 29	22 4 50	+1 46·15	−5 18·4	12 : 12	12 42 54·70	9·257	−17 29 37·5	0·850	2
,, 30	19 20 5	+1 14·12	−1 31·1	12 : 12	12 42 42·64	9·168*n*	−15 41 49·6	0·843	3
May 4	20 57 16	+0 16·19	+4 21·3	22 : 11	12 42 13·66	8·910	− 8 14 20·9	0·800	4
,, 9	20 32 34	+0 30·42	+4 30·1	12 : 12	12 42 33·00	8·843	− 0 52 45·5	0·740	5
			Comet Carrasco (1932 *c*)						
Apr. 26	20 15 2	+2 35·65	−1 59·7	12 : 12	12 11 8·80	7·785*n*	+23 26 20·2	0·338	6
,, 27	19 34 0	+2 9·30	+8 16·7	5 : 5	12 10 3·72	8·930*n*	+23 6 36·0	0·355	7
,, 29	19 56 28	−1 56·52	+1 34·6	12 : 12	12 7 54·38	8·138*n*	+22 25 49·8	0·367	8

Mean Positions of Comparison Stars for 1932·0

Mag.	α	δ	Authorities
	h m s	° ′ ″	
8·2	12 42 15·65	−24 9 52·8	Cordoba A 9565
8·0	12 41 8·55	−17 24 19·1	A.G. Wash. 4878
8·6	12 41 28·52	−15 40 18·5	,, ,, 4880
8·7	12 41 57·47	− 8 18 42·2	,, W.-Ott. 4634

* Mag.	α	δ	Authorities
	h m s	° ′ ″	
5 9·0	12 42 2·58	− 0 57 15·6	A.G. Nicol 3462
6 8·5	12 8 33·15	+23 28 19·9	,, Berlin B 4462
7 7·5	12 7 54·42	+22 58 19·3	,, ,, ,, 4458
8 7·9	12 9 50·90	+22 24 15·2	,, ,, ,, 4467

Remarks

(a) *Comet Houghton-Ensor.*

1932 April 25.—Diffused round nebulosity of about 65″ diameter with a slight condensation around a central stellar nucleus of nearly 11 mag.

April 26.—The coma looks much more extended (diameter about 150″), while the nucleus is hardly visible (about 13 mag.).

April 29.—From time to time a faint nucleus of 14 mag. is visible in the middle of a slight condensation. Diameter of the coma 125″ approximately.

April 30.—Same appearance. The measurements were taken on the centre of the coma. Its diameter is about 110″.

May 3.—The coma looks larger. Diameter nearly 150″.

May 4.—Observation difficult. Sky not so clear.

May 9.—Diameter of the coma about 130″. As usual the comet is visible in the finder (80 mm.), and its motion in declination is very appreciable.

(b) *Comet Carrasco.*

1932 April 26.—Round nebulosity with ill-defined boundaries and a slight condensation around a stellar nucleus of about 12 mag., which from time to time looks a little eccentric towards N.W. Diameter of the coma about 60″. Appreciable motion in declination.

April 27.—Observation interrupted by clouds.

April 29.—The comet looks a little fainter. Nucleus of about 13 mag. Diameter of the coma nearly 60″.

National Observatory of Athens :
1932 *May* 17.

A PRELIMINARY DISCUSSION OF LATITUDE VARIATION AT DEHRA DUN.

G. P. Rao, M.A.

Preliminary values of the variation of latitude at Dehra Dun during 1930, in the sense, observed *minus* mean latitude, are given in the *Survey of India Geodetic Report*, 7, 28, 1930–31. These values, when plotted (see fig. 1), indicate a fairly well-defined annual periodic variation. Analysing the results in the form

$$a_0 + (a_1 \cos t + b_1 \sin t) + (a_2 \cos 2t + b_2 \sin 2t),$$

where t is the interval of time reckoned from January $1^d\ 0^h$ at 30° per month, the following values of the Fourier constants are obtained :—

$$a_0 = +0''{\cdot}004, \qquad a_1 = +0''{\cdot}242, \qquad a_2 = -0''{\cdot}013,$$
$$b_1 = -0''{\cdot}064, \qquad b_2 = -0''{\cdot}069,$$

so that

$$\sqrt{(a_1^2+b_1^2)} = +0''\cdot250, \qquad \sqrt{(a_2^2+b_2^2)} = +0''\cdot070,$$
$$\tan^{-1}(b_1/a_1) = -14^\circ\ 49', \qquad \tan^{-1}(b_2/a_2) = -100^\circ\ 40'.$$

Therefore the variation of latitude at Dehra Dun can be represented by the formula

$$\delta\phi = +0''\cdot004 + 0''\cdot250\cos(t+15^\circ) + 0''\cdot070\cos(2t+100^\circ).$$

The annual component with amplitude $0''\cdot25$ and phase 15° represents the motion of the pole. The semi-annual component has a small amplitude, and may be due partly to refraction anomalies and partly to systematic errors in the declinations of stars. The smooth curve drawn in fig. I represents the above formula. The following table shows the residuals between the observed values and those computed from the formula :—

TABLE I

Date	Observed	Computed	O. – C.
	"	"	"
1930·125	+0·15	+0·06	+0·09
·208	– ·07	– ·02	– ·05
·292	– ·06	– ·08	+ ·02
·375	– ·15	– ·14	– ·01
·458	– ·25	– ·22	– ·03
·542	– ·19	– ·26	+ ·07
·625	– ·21	– ·19	– ·02
·708	– ·10	– ·02	– ·08
·792	+ ·26	+ ·17	+ ·09
·875	+ ·29	+ ·29	·00
·958	+ ·25	+ ·28	– ·03
1931·042	+ ·13	+ ·18	– ·05

The residuals given in the last column show that the formula agrees with observations to within $\pm 0''\cdot1$. The variation of latitude at Dehra Dun is somewhat larger than the corresponding variation at Greenwich (see fig. II). The maximum and minimum of the latitude variation curve at Dehra Dun follow those of Greenwich by about three months.

The results do not cover a sufficient period to give much reliable information regarding the Kimura term in latitude variation, which is of obscure origin. Using the International x and y co-ordinates of polar motion for 1930 (see *Bulletin Géodésique*, **30**, 160, 1931) and computing by the formula

$$\delta\phi = x\cos\lambda + y\sin\lambda,$$

a direct comparison with the Dehra Dun results is obtained, and is shown in Table II.

The results agree fairly well in phase, but not so well in amplitude. The variation at Dehra Dun is somewhat greater than that obtained from the International Latitude Service Observations, but against this it must be noted that the number of pairs contributing to each mean is considerably

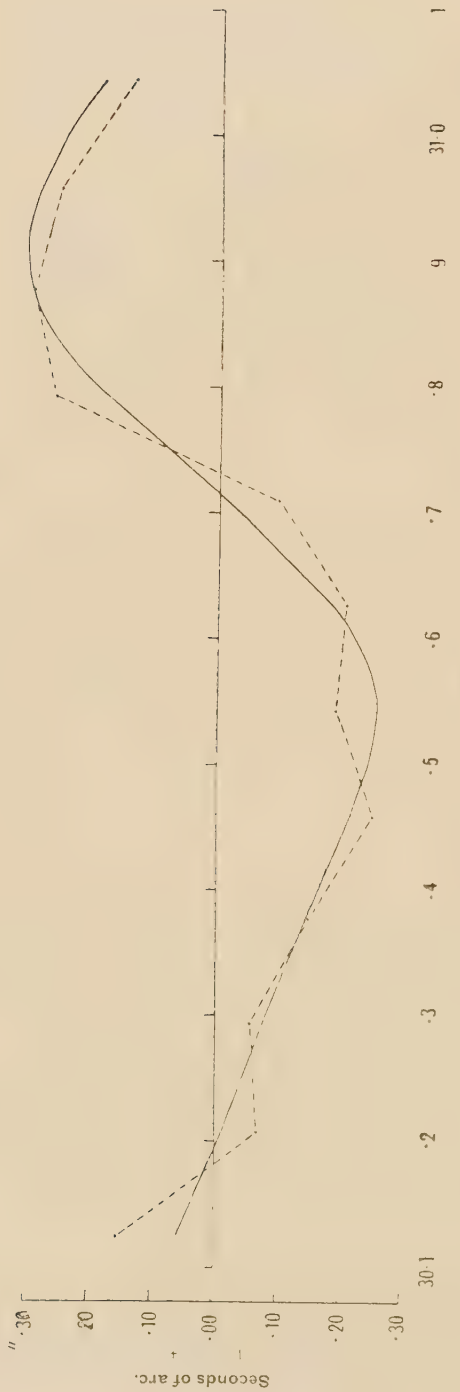


Fig. I Latitude Variation 1930. Dehra Dün.

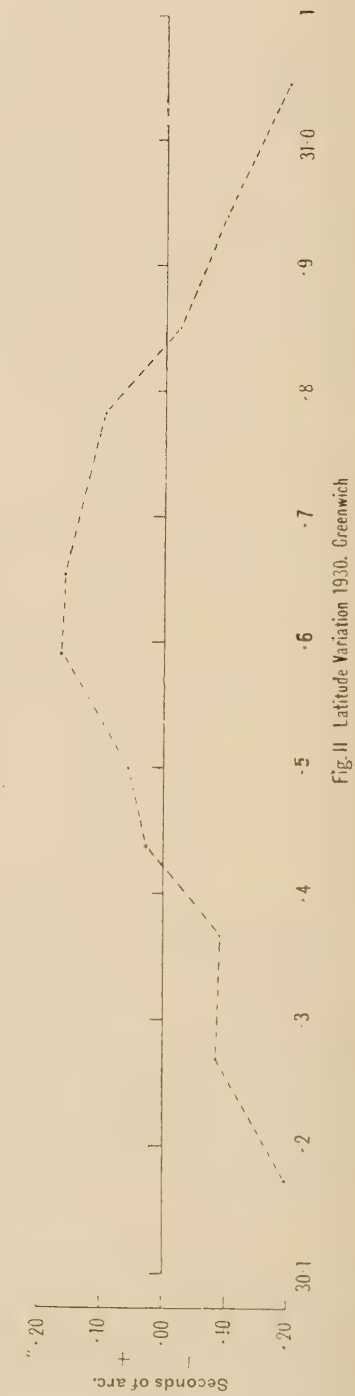


Fig. II Latitude Variation 1930. Greenwich

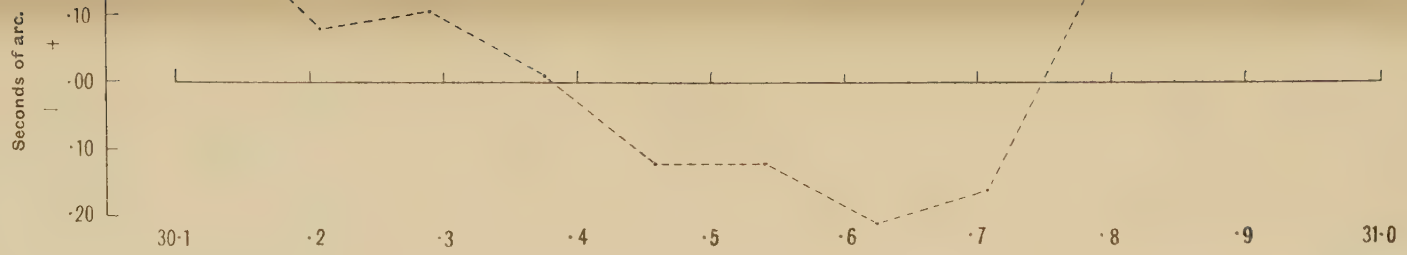


Fig. III Z-Term in Latitude Variation 1930. Dehra Dūn.

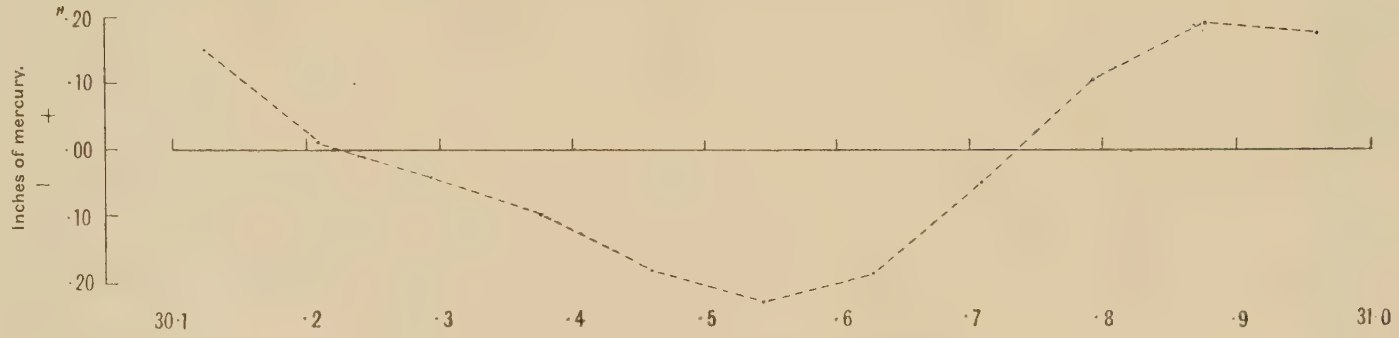


Fig. IV Pressure Variation 1930. Dehra Dūn.

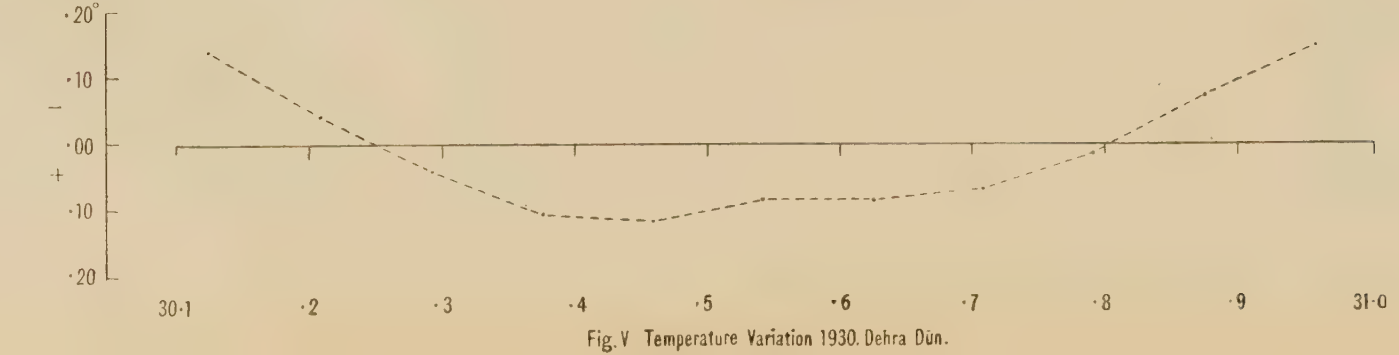


Fig. V Temperature Variation 1930. Dehra Dūn.

less, and the group corrections are not nearly so well determined. The residuals given in the last column of Table II may be regarded as representing the Z-term at Dehra Dun. These residuals, when plotted (see fig. III), show systematic annual variation, with deviations more or less of local character. Analysis gives $Z = 0''{\cdot}02 + 0''{\cdot}17 \cos(t - 25°)$.

TABLE II

Date	Observed Variation at Dehra Dun	Computed Variation from International x and y	Residuals. O. – C.
	″	″	″
1930·125	+0·15	−0·09	+0·24
·208	− ·07	− ·15	+ ·08
·292	− ·06	− ·17	+ ·11
·375	− ·15	− ·16	+ ·01
·458	− ·25	− ·13	− ·12
·542	− ·19	− ·07	− ·12
·625	− ·21	·00	− ·21
·708	− ·10	+ ·06	− ·16
·792	+ ·26	+ ·09	+ ·17
·875	+ ·29	+ ·08	+ ·21

The amplitude of the Z-term is unusually large. The variation of Z-term at Dehra Dun agrees fairly well in phase with the variation of pressure (see fig. IV) and the variation of temperature (see fig. V) at Dehra Dun. The Z-term in latitude variation is therefore probably dependent in some way upon the local meteorological conditions.

OCCULTATIONS OBSERVED AT THE ROYAL OBSERVATORY, CAPE OF GOOD HOPE, IN THE YEARS 1923–1931.

H. Spencer Jones, F.R.S., and A. J. Wilkin.

The observations of occultations of stars at the Cape Observatory during the period 1923–31 have been restricted mainly to occultations of bright stars predicted at the Nautical Almanac Office. The observed and tabular quantities are given in Table I. The first column of this table gives the serial number of the occultation ; the second gives the date of observation ; the third and fourth columns give the name and magnitude of the occulted star. The fifth column contains the phase observed, where

D.D. = disappearance at dark limb.
R.D. = reappearance at dark limb.
D.B. = disappearance at bright limb.
R.B. = reappearance at bright limb.

The instrument with which the observation was made is noted in the

sixth column. The following are the particulars for the three instruments used :—

Instrument	Longitude	Latitude	Log $\rho \cos \phi'$	Log $\rho \sin \phi'$
	h m s	° ′ ″		
10-inch equatorial	−1 13 54·80	−33 56 7·8	9·91936	−9·74436
7-inch ,,	54·58	1·4	7	4
6-inch ,,	54·74	7·0	6	6

The longitudes and latitudes have been referred to the 8-inch transit circle, for which the adopted data are :

Longitude	$-1^h\ 13^m\ 54^s{\cdot}70$
Latitude	$-33°\ 56'\ 3''{\cdot}4$
ϕ'	$-33°\ 45'\ 19''{\cdot}8$
$\log \rho$	9·9995467.

The seventh column contains the initials of the observers, as follows :—

Mr. W. H. Cox	C.	Dr. J. Lunt	L.
Mr. L. T. Davis	L.D.	Miss C. Orpen	C.O.
Dr. J. K. E. Halm	J.H.	Mr. J. A. J. Pead	A.P.
Mr. H. Horrocks	H.H.	Mr. J. H. Peirce	H.P.
Mr. J. W. Jackson	J.J.	Mr. A. J. Wilkin	A.W.
Dr. H. Spencer Jones	J.	Mr. R. Woodgate	R.W.

The eighth and ninth columns give respectively the Cape sidereal time of the observed phenomenon and the corresponding Greenwich mean time (reckoned from midnight).

The portion of Table I on the right-hand page gives the tabular quantities, which are self-explanatory. The star places have been adopted throughout from the *Nautical Almanac*, except in a few cases of occultations of fainter stars. The sources of the places for these stars are noted at the end of the table. The right ascension and declination of the Moon are interpolated from the *Nautical Almanac* positions, based upon Brown's tables of the Moon, but with corrections which correspond to the following provisional corrections for each year to the mean longitude according to the tables :—

Year	$\Delta\lambda_m$	Year	$\Delta\lambda_m$
	″		″
1923	+7·5	1928	+6·5
24	7·2	29	5·8
25	7·0	30	4·6
26	6·7	31	5·0
27	6·6		

The effect of these corrections is to make the final residuals sufficiently small for the additional corrections to the mean and true longitudes to be considered identical. For the year 1923 an additional correction was made to the Moon's place, of +0″·08 cosine mean anomaly (vide *Nautical Almanac*, 1923, p. v).

Notes to some of the observations are given at the end of Table I.

TABLE I

Observed Quantities

No.	Date	Star	Mag.	Phase	Instrument	Obs.	Cape Sid. Time	G.M.T.
	1923						h m s	h m s
1	Jan. 20	252 B Aquarii	5·8	D.D.	7-inch	A.P.	4 33 17·85	19 22 48·61
2	,, 25	38 Arietis	5·2	D.D.	7 ,,	A.P.	3 58 3·92	18 28 0·90
3	Aug. 8	119 Tauri	4·9	R.D.	7 ,,	C.	1 53 36·08	3 37 11·30
4	,, 20	125 B Ophiuchi	6·2	D.D.	7 ,,	C.	17 37 30·55	18 31 20·24
5	Sept. 14	ξ^2 Libræ	5·6	D.D.	7 ,,	C.	19 40 17·05	18 55 28·95
6	,, 15	203 B Libræ	6·2	D.D.	7 ,,	C.	20 15 29·86	19 26 40·09
7	,, 16	78 B Ophiuchi	6·5	D.D.	7 ,,	C.	20 45 48·08	19 52 57·44
8	Oct. 17	31 B Capricorni	6·4	D.D.	7 ,,	C.	21 48 43·58	18 53 49·55
9	,, 18	18 Aquarii	5·5	D.D.	7 ,,	C.	23 37 2·29	20 37 54·61
10	Dec. 26	π Cancri	5·6	R.D.	7 ,,	C.	5 10 22·14	21 39 2·08
11	,, 27	α Leonis	1·3	R.D.	7 ,,	C.	5 44 14·11	22 8 52·59
12	,, 29	χ Leonis	4·7	R.D.	7 ,,	C.	8 54 52·14	1 15 3·48
	1924							
13	Feb. 1	164 B Ophiuchi	6·0	R.D.	7 ,,	C.	12 10 16·18	2 16 14·55
14	,, 7	342 B Aquarii	6·5	D.D.	7 ,,	J.	5 2 42·73	18 42 19·78
15	,, 7	,,	,,	,,	6 ,,	J.H.	42·49	19·38
16	,, 14	m Tauri	5·0	D.D.	7 ,,	J.	6 32 36·41	19 44 27·38
17	,, 26	48 Libræ	4·6	R.D.	6 ,,	J.H.	9 58 31·33	22 22 37·52
18	Apr. 10	68 Orionis	5·7	D.D.	6 ,,	C.	9 45 22·85	19 16 31·32
19	,, 10	B.D. +19° 1254	8·1	D.D.	6 ,,	C.	9 47 7·55	19 18 15·73
20	,, 22	φ Ophiuchi	4·4	R.D.	6 ,,	C.	20 2 25·30	4 48 37·71
21	July 5	ψ Leonis	5·6	D.D.	6 ,,	C.	14 5 31·10	17 57 48·73
22	,, 14	305 B Ophiuchi	6·3	D.D.	6 ,,	C.	13 41 39·66	16 58 38·00
23	,, 18	e Aquarii	5·4	R.D.	6 ,,	J.H.	20 4 26·85	23 4 38·84
24	,, 19	h Aquarii	5·4	R.D.	6 ,,	J.H.	20 13 46·33	23 10 0·88
25	Aug. 4	b Virginis	5·2	D.D.	6 ,,	C.	14 39 23·25	16 33 38·04
26	,, 6	566 B Virginis	6·4	D.D.	6 ,,	C.	15 24 49·45	17 11 4·98
27	,, 9	φ Ophiuchi	4·4	D.D.	6 ,,	C.	15 11 12·64	16 45 42·66
28	,, 12	267 B Sagittarii	5·8	D.D.	6 ,,	C.	15 40 27·44	17 3 4·95
29	,, 20	39 B Arietis	6·5	R.D.	6 ,,	C.	23 31 20·18	0 25 9·19
30	Oct. 3	125 B Ophiuchi	6·2	D.D.	6 ,,	C.	22 45 7·87	20 42 8·66
31	Nov. 4	151 B Capricorni	6·1	D.D.	6 ,,	C.	2 4 34·27	21 55 13·36
32	Dec. 4	4 Ceti	6·3	D.D.	6 ,,	C.	1 32 33·60	19 25 20·64
33	,, 4	5 Ceti	6·3	D.D.	6 ,,	C.	1 52 32·30	19 45 16·07
	1925							
34	Apr. 13	90 B Ophiuchi	6·5	R.D.	6 ,,	C.	18 15 47·22	3 38 37·67
35	,, 29	79 Geminorum	6·3	D.D.	6 ,,	C.	9 53 11·06	18 10 33·41
36	May 13	f Sagittarii	5·1	R.D.	6 ,,	L.	19 58 31·86	3 23 8·23
37	,, 13	v Capricorni	5·3	R.D.	6 ,,	L.	15 50 15·97	23 11 37·10
38	,, 15	45 Capricorni	5·8	R.D.	6 ,,	L.	19 9 29·84	2 26 22·43
39	,, 20	39 B Arietis	6·5	R.D.	6 ,,	L.	21 58 9·39	4 54 54·79
40	Aug. 1	16 Sagittarii	5·9	D.D.	6 ,,	L.	19 55 35·63	22 1 43·72
41	,, 3	σ Capricorni	5·5	D.D.	6 ,,	L.	16 42 54·52	18 41 42·36
42	,, 25	ξ^2 Libræ	5·6	D.D.	6 ,,	L.	19 0 55·90	19 32 51·15

TABLE I

Tabular Quantities

No.	Star's Apparent R.A.	Star's Apparent Dec.	Moon's R.A.	Moon's Dec.	Moon's Hor. Eq. Parallax	From N. Point: Disappearance χ	From N. Point: Direction of Motion P
	h m s	° ′ ″	h m s	° ′ ″	″	° ′	° ′
1	22 51 10·41	− 5 23 55·3	22 53 56·26	− 6 8 10·9	3287·17	16 6	72 38
2	2 40 46·10	+12 7 15·8	2 40 46·91	+11 29 58·4	3277·63	95 11	75 36
3	5 27 42·84	+18 32 11·1	5 25 56·21	+18 3 46·7	3329·52	230 52	85 42
4	17 3 48·87	−17 30 23·1	17 3 47·88	−17 32 45·3	3518·90	152 32	96 2
5	14 52 36·42	−11 5 57·3	14 54 57·98	−11 45 51·1	3605·70	51 30	104 24
6	15 52 14·82	−14 36 13·0	15 54 14·95	−15 1 27·5	3577·27	95 46	100 57
7	16 51 36·98	−16 40 58·6	16 53 42·70	−17 15 54·0	3542·50	47 16	96 47
8	20 24 25·71	−15 59 33·3	20 24 43·65	−16 8 52·9	3420·42	125 2	80 35
9	21 20 1·67	−13 12 15·4	21 20 54·98	−13 26 15·2	3374·99	119 36	76 58
10	9 11 2·68	+15 15 19·6	9 9 17·07	+14 41 59·6	3380·86	258 46	102 11
11	10 4 19·77	+12 20 15·3	10 2 5·49	+11 33 13·4	3413·21	320 7	105 11
12	11 1 5·81	+ 7 44 47·6	11 0 6·75	+ 7 19 25·9	3451·71	221 21	107 34
13	17 15 26·81	−17 40 32·5	17 12 55·59	−17 57 36·5	3570·52	228 10	96 9
14	23 27 34·85	− 4 30 16·0	23 29 45·88	− 5 7 36·5	3383·85	64 51	71 45
15	...	...	...	...	...	...	...
16	5 2 57·93	+18 32 31·5	5 3 26·29	+17 40 0·9	3269·58	44 56	83 6
17	15 53 56·11	−14 3 36·4	15 51 22·94	−14 45 49·2	3557·66	310 55	101 59
18	6 7 31·12	+19 48 22·9	6 9 13·88	+19 3 1·2	3264·99	57 20	88 15
19	6 7 33·21	+19 31 59·1	6 9 17·53	+19 3 2·8	3265·03	123 48	88 15
20	16 26 48·71	−16 26 50·4	16 30 47·15	−16 47 39·0	3644·78	259 40	99 28
21	9 39 35·63	+14 22 7·3	9 41 27·90	+13 48 24·3	3342·62	93 12	103 56
22	17 51 29·34	−18 47 15·2	17 47 15·47	−19 4 55·3	3648·00	113 24	94 9
23	22 6 35·95	−11 56 4·9	22 5 59·25	−12 16 55·7	3525·67	261 29	74 44
24	23 1 13·99	− 8 6 0·2	22 59 47·01	− 8 21 2·0	3471·64	222 1	72 19
25	11 56 3·76	+ 4 4 41·0	11 57 10·68	+ 3 36 27·6	3437·67	112 48	109 9
26	13 39 57·74	− 5 7 0·2	13 40 33·83	− 5 25 30·5	3499·47	128 28	108 44
27	16 26 49·18	−16 26 48·9	16 25 32·09	−16 29 14·5	3577·30	171 21	100 31
28	19 32 41·56	−18 23 50·1	19 29 14·53	−19 1 51·3	3595·52	24 13	85 47
29	2 0 52·38	+ 7 22 25·9	1 59 32·88	+ 6 59 55·5	3323·09	212 54	71 47
30	17 3 51·20	−17 30 28·5	17 6 37·13	−18 13 32·8	3573·87	37 40	97 30
31	21 45 36·79	−13 4 24·4	21 48 16·89	−13 46 6·2	3474·19	17 1	75 30
32	0 3 52·94	− 2 58 3·2	0 4 5·62	− 3 21 58·2	3388·85	109 29	70 12
33	0 4 21·03	− 2 51 58·3	0 4 45·82	− 3 18 21·4	3388·27	100 36	70 11
34	16 55 22·92	−18 7 56·5	16 57 40·34	−18 29 6·4	3563·97	285 23	99 0
35	7 40 44·94	+20 29 45·3	7 42 5·11	+20 1 56·1	3253·95	157 1	95 0
36	19 42 0·15	−19 56 26·4	19 43 22·90	−20 6 54·8	3589·38	256 26	84 55
37	20 35 47·48	−18 24 6·0	20 33 14·12	−18 38 2·3	3568·50	220 20	80 47
38	21 39 55·62	−15 5 33·2	21 38 55·71	−15 22 33·4	3534·10	253 51	75 57
39	2 0 52·72	+ 7 22 27·6	1 59 10·34	+ 6 49 54·5	3368·40	265 9	70 21
40	18 10 47·53	−20 24 35·4	18 11 14·17	−20 36 22·1	3641·17	104 5	93 22
41	20 15 6·54	−19 21 0·8	20 11 4·38	−19 39 17·9	3678·23	102 9	82 51
42	14 52 42·58	−11 6 27·4	14 55 4·07	−11 19 33·8	3440·59	151 50	107 42

No.	Date	Star	Mag.	Phase	Instrument	Obs.	Cape Sid. Time	G.M.T.
	1925						h m s	h m s
43	Aug. 25	B.D. −10° 3992	9·0	D.D.	6-inch	L.	19 30 43·69	20 2 34·05
44	,, 25	17 Libræ	6·4	D.D.	6 ,,	L.	19 33 9·39	20 4 59·36
45	,, 25	18 Libræ	5·9	D.D.	6 ,,	L.	19 59 35·89	20 31 21·52
46	,, 29	29 Sagittarii	5·3	D.D.	6 ,,	L.	19 58 27·56	20 14 29·76
47	Sept. 5	ν Piscium	4·7	R.D.	6 ,,	L.	21 52 32·16	21 40 44·32
48	Oct. 24	B.D. −19° 5777	8·7	D.D.	6 ,,	L.	23 58 26·28	20 33 38·38
49	,, 24	σ Capricorni	5·5	D.D.	6 ,,	L.	23 59 42·08	20 34 53·97
50	Nov. 2	68 Tauri	4·3	R.D.	6 ,,	L.	0 48 7·13	20 47 47·91
51	,, 22	ι Aquarii	4·4	D.D.	6 ,,	L.	3 41 44·81	22 22 18·97
52	Dec. 5	η Cancri	5·5	R.D.	6 ,,	L.	6 11 35·59	0 4 34·26
53	,, 24	ξ' Ceti	4·5	D.D.	6 ,,	L.	5 41 28·18	22 15 53·55
54	,, 28	ζ Tauri	3·0	D.D.	6 ,,	L.	5 16 49·78	21 35 35·54
	1926							
55	Jan. 22	30 B Tauri	6·4	D.D.	6 ,,	L.	5 6 12·16	19 46 41·88
56	Feb. 17	25 Arietis	6·5	D.D.	6 ,,	J.H.	5 21 50·59	18 20 4·11
57	Apr. 17	η Geminorum	3·3	D.D.	6 ,,	J.H.	10 16 48·21	19 22 14·90
58	May 2	o Sagittarii	3·9	R.D.	6 ,,	J.H.	14 52 31·78	22 58 14·68
	1927							
59	Jan. 11	ξ Arietis	5·5	D.D.	7 ,,	J.	4 53 8·75	20 17 53·01
60	,, 14	105 Tauri	6·0	D.D.	10 ,,	J.	3 34 59·79	18 48 8·90
61	,, 14	108 Tauri	6·2	D.D.	7 ,,	J.	8 38 57·69	23 51 17·22
62	,, 15	1 Geminorum	4·3	D.D.	6 ,,	J.	3 26 50·64	18 36 5·23
63	Apr. 7	5 Geminorum	5·9	D.D.	7 ,,	J.	8 16 21·99	18 2 24·83
64	,, 7	,,	,,	,,	10 ,,	H.P.	22·31	24·93
65	,, 8	48 Geminorum	5·8	D.D.	10 ,,	R.W.	10 32 40·05	20 14 24·44
66	June 5	η Leonis	3·6	D.D.	7 ,,	J.	14 4 23·13	19 57 30·31
67	Oct. 14	ω Tauri	4·8	R.D.	6 ,,	H.H.	5 35 18·33	2 54 44·61
	1928							
68	Jan. 14	566 B Virginis	6·4	R.D.	6 ,,	H.H.	8 43 35·90	23 56 51·63
69	,, 31	247 B Tauri	5·8	D.D.	7 ,,	J.	7 6 37·37	21 13 18·65
70	,, 31	,,	,,	,,	6 ,,	H.H.	36·93	18·05
71	Mar. 17	308 B Sagittarii	6·3	R.D.	6 ,,	H.H.	15 2 46·21	2 11 13·45
72	Apr. 9	β^1 Scorpii	2·9	D.B.	6 ,,	J.	17 15 41·01	2 53 20·62
73	,, 9	β^2 Scorpii	5·0	R.D.	6 ,,	J.	18 35 12·11	4 12 38·69
74	,, 9	β^1 Scorpii	2·9	R.D.	6 ,,	J.	18 35 32·01	4 12 58·53
75	,, 23	118 Tauri	5·4	D.D.	10 ,,	J.	8 50 25·78	17 30 29·47
76	May 6	ω Ophiuchi	4·5	R.D.	7 ,,	J.	13 53 48·61	21 41 56·00
77	,, 6	,,	,,	,,	6 ,,	H.H.	49·87	57·10
78	,, 7	44 Ophiuchi	4·1	D.B.	7 ,,	J.	14 44 20·84	22 28 24·04
79	,, 7	,,	,,	R.D.	7 ,,	J.	15 59 15·24	23 43 6·17
80	,, 7	,,	,,	,,	6 ,,	H.H.	15·00	5·77
81	,, 9	λ Sagittarii	2·9	D.B.	6 ,,	H.H.	20 17 58·83	3 57 11·31
82	,, 10	ψ Sagittarii	4·8	R.D.	6 ,,	H.H.	16 54 47·87	0 30 37·72
83	,, 12	χ Capricorni	5·3	R.D.	6 ,,	H.H.	19 30 30·33	2 58 2·85
84	June 29	β^1 Scorpii	2·9	D.D.	10 ,,	H.H.	17 59 11·18	22 14 18·93
85	,, 29	β^2 Scorpii	5·0	D.D.	10 ,,	H.H.	18 0 50·08	22 15 57·56
86	July 8	ψ^3 Aquarii	5·2	R.D.	6 ,,	H.H.	0 51 52·48	4 34 25·38

No.	Star's Apparent R.A.	Star's Apparent Dec.	Moon's R.A.	Moon's Dec.	Moon's Hor. Eq. Parallax	From N. Point: Disappearance	From N. Point: Direction of Motion
	h m s	° ′ ″	h m s	° ′ ″	″	° ′	° ′
43	14 54 6·22	−11 1 56·1	14 56 7·87	−11 24 32·9	3441·41	110 3	107 40
44	14 54 10·24	−10 51 14·7	14 56 13·07	−11 24 57·3	3441·49	69 6	107 39
45	14 54 50·91	−10 50 35·7	14 57 9·74	−11 29 22·0	3442·23	51 44	107 36
46	18 45 15·32	−20 24 33·0	18 46 8·55	−20 55 18·8	3604·34	11 24	90 13
47	1 37 33·78	+ 5 6 43·1	1 35 56·08	+ 4 31 23·6	3491·65	272 43	69 30
48	20 15 5·93	−19 21 59·9	20 17 4·79	−19 53 42·0	3554·40	52 15	82 6
49	20 15 5·74	−19 21 4·0	20 17 7·92	−19 53 35·9	3554·41	48 47	82 5
50	4 21 11·92	+17 45 31·2	4 19 3·62	+17 21 4·4	3365·21	206 3	77 11
51	22 2 25·02	−14 13 52·2	22 4 50·20	−14 36 7·0	3545·91	121 27	73 57
52	8 28 25·39	+20 41 34·5	8 27 33·10	+19 53 20·0	3251·28	298 5	98 42
53	2 9 4·19	+ 8 29 54·9	2 11 4·11	+ 7 41 16·7	3411·49	31 21	69 41
54	5 33 13·60	+21 5 50·9	5 32 6·67	+20 29 24·4	3298·19	124 11	82 46
55	3 33 38·25	+15 11 14·8	3 33 52·13	+14 26 41·1	3363·72	77 30	73 40
56	2 23 26·14	+ 9 52 6·1	2 25 12·40	+ 9 0 39·6	3475·78	28 32	70 9
57	6 10 23·90	+22 31 43·8	6 12 13·67	+21 54 49·6	3337·87	90 18	86 24
58	19 0 15·56	−21 51 1·1	18 58 13·66	−22 18 42·8	3518·98	292 16	89 19
59	2 20 53·49	+10 16 45·5	2 23 6·09	+ 9 22 16·0	3510·06	350 3	68 31
60	5 3 33·59	+21 36 33·2	5 1 10·41	+20 51 51·2	3429·48	93 53	79 17
61	5 11 4·49	+22 12 8·3	5 12 54·99	+21 21 24·0	3423·44	48 16	80 20
62	5 59 41·30	+23 16 6·0	5 56 45·59	+22 42 58·3	3400·85	132 37	84 18
63	6 7 3·03	+24 26 16·6	6 8 1·48	+23 30 41·6	3441·82	50 35	85 45
64	...	...	...	...	...	...	...
65	7 8 0·00	+24 15 9·6	7 10 4·26	+23 47 35·2	3384·29	148 22	92 0
66	10 3 21·09	+17 7 12·0	10 5 9·44	+16 37 2·9	3310·07	112 52	107 49
67	4 13 1·33	+20 24 9·3	4 14 32·00	+19 52 30·7	3605·75	193 21	74 55
68	13 40 8·32	− 5 8 9·7	13 38 7·56	− 5 43 29·0	3256·44	295 47	114 23
69	4 23 44·05	+21 27 38·5	4 24 53·74	+20 47 9·0	3540·34	102 54	75 43
70	...	...	...	...	...	...	...
71	19 49 58·13	−24 7 23·1	19 47 48·67	−24 32 21·6	3399·53	278 42	83 22
72	16 1 15·53	−19 36 40·2	16 1 14·85	−19 53 20·4	3245·98	79 33	106 27
73	16 1 16·00	−19 36 26·2	16 3 54·63	−20 4 20·6	3246·44	321 4	106 13
74	16 1 15·53	−19 36 40·2	16 3 55·30	−20 4 23·3	3246·44	319 56	106 13
75	5 24 49·46	+25 5 39·7	5 26 28·89	+24 20 46·3	3602·89	84 50	81 36
76	16 27 53·24	−21 18 55·1	16 26 54·49	−21 37 31·3	3246·29	283 25	104 18
77	,,	,,	54·53	31·5	,,	,,	,,
78	17 21 59·46	−24 6 42·9	17 19 12·41	−24 8 29·4	3259·27	140 19	99 12
79	,,	,,	17 21 53·60	−24 14 21·1	3260·10	259 44	98 56
80	...	...	...	...	...	...	...
81	18 23 32·71	−25 27 49·6	18 24 2·84	−25 38 17·3	3283·74	89 15	92 21
82	19 11 8·48	−25 22 56·9	19 10 20·58	−25 36 31·5	3307·12	278 24	87 20
83	21 4 26·67	−21 29 1·5	21 4 10·39	−21 38 33·9	3387·48	255 27	75 44
84	16 1 16·54	−19 36 42·3	16 2 8·01	−20 2 39·1	3251·17	43 2	106 29
85	16 1 17·01	−19 36 28·3	16 2 11·33	−20 2 52·9	3251·18	40 51	106 29
86	23 15 14·34	−10 0 7·9	23 17 12·87	−10 11 34·9	3453·96	217 42	66 14

No.	Date	Star	Mag.	Phase	Instrument	Obs.	Cape Sid. Time	G.M.T.
	1928						h m s	h m s
87	July 30	ψ Sagittarii	4·8	D.D.	6-inch	H.H.	16 15 32·93	18 29 4·46
88	Aug. 29	κ Capricorni	4·8	D.D.	6 ,,	H.H.	19 11 14·36	19 26 19·84
89	Sept. 23	*h* Sagittarii	4·7	D.D.	6 ,,	H.H.	0 42 24·36	23 18 17·89
	1929							
90	Mar. 4	θ Ophiuchi	3·3	R.D.	6 ,,	H.H.	14 56 24·24	2 56 56·30
91	,, 29	28 Libræ	6·2	R.D.	10 ,,	L.D.	16 18 11·32	2 40 12·23
92	Apr. 16	ψ Cancri	5·9	D.D.	10 ,,	J.	9 21 20·66	18 29 47·62
93	,, 16	,,	,,	,,	6 ,,	H.H.	20·80	47·82
94	June 18	26 Libræ	6·3	D.D.	6 ,,	H.H.	13 19 28·36	18 19 33·98
95	,, 18	28 Libræ	6·2	D.D.	6 ,,	H.H.	17 57 41·07	22 57 1·11
96	Aug. 11	8 Libræ	5·4	D.D.	6 ,,	A.W.	19 59 10·24	21 25 51·15
97	,, 11	α Libræ	2·7	D.D.	6 ,,	A.W.	20 6 54·54	21 33 34·19
98	,, 13	ρ Ophiuchi	4·7	D.D.	10 ,,	H.P.	17 20 0·56	18 39 15·67
99	Oct. 14	29 Aquarii	6·5	D.D.	6 ,,	H.H.	2 54 54·61	0 12 45·21
100	Dec. 18	ι Geminorum	3·8	D.B.	6 ,,	J.J.	8 55 19·09	1 56 36·43
	1930							
101	Mar. 5	40 Arietis	6·0	D.D.	6 ,,	H.H.	6 12 6·15	18 7 9·17
102	,, 13	η Leonis	3·6	D.D.	6 ,,	H.H.	13 41 33·24	1 7 51·27
103	,, 15	γ Virginis *N.p.*	3·7	R.D.	6 ,,	H.H.	9 41 7·51	20 56 17·21
104	,, 15	γ Virginis *S.f.*	3·7	R.D.	6 ,,	H.H.	9 41 18·91	20 56 28·58
105	Apr. 24	74 Aquarii	5·8	R.D.	10 ,,	H.H.	19 7 40·45	3 47 56·86
106	May 9	10 Virginis	6·2	D.D.	6 ,,	H.H.	17 4 51·24	0 46 29·18
107	,, 9	38 Virginis	6·1	D.D.	6 ,,	H.H.	13 9 14·35	20 47 34·99
108	Nov. 1	χ Aquarii	5·3	D.D.	10 ,,	J.	22 52 8·28	18 56 53·16
109	,, 1	,,	,,	,,	6 ,,	C.O.	8·22	53·16
110	,, 8	38 B Aurigæ	6·5	R.D.	6 ,,	H.H.	2 47 31·83	22 24 6·84
111	Dec. 5	27 Tauri	3·7	D.D.	6 ,,	H.H.	6 48 40·16	0 42 21·96
112	,, 5	28 Tauri	5·2	D.D.	6 ,,	H.H.	6 54 37·96	0 48 18·78
	1931							
113	Jan. 7	46 Leonis	5·8	R.D.	10 ,,	H.H.	6 56 22·64	22 36 22·07
114	,, 28	23 Tauri	4·3	D.D.	6 ,,	H.H.	4 16 15·45	18 34 7·02
115	,, 28	20 Tauri	4·1	D.D.	6 ,,	H.H.	4 18 29·05	18 36 20·25
116	,, 28	W.Z.C. 224	8·1	D.D.	10 ,,	J.	4 51 2·91	19 8 48·72
117	,, 28	,,	,,	,,	6 ,,	H.H.	2·25	48·12
118	,, 28	B.D. +23° 540	6·8	D.D.	10 ,,	J.	4 57 10·51	19 14 55·31
119	,, 28	,,	,,	,,	6 ,,	H.H.	10·85	55·71
120	,, 28	η Tauri	3·0	D.D.	10 ,,	J.	4 58 30·81	19 16 15·40
121	,, 28	,,	,,	,,	6 ,,	H.H.	31·05	15·70
122	,, 28	B.D. +24° 562	6·7	D.D.	10 ,,	J.	5 22 36·71	19 40 17·35
123	,, 28	,,	,,	,,	6 ,,	H.H.	36·15	16·85
124	,, 28	W.Z.C. 227	6·6	D.D.	10 ,,	J.	5 37 21·11	19 54 59·33
125	,, 28	,,	,,	,,	6 ,,	H.H.	20·85	59·13
126	Feb. 3	η Leonis	3·6	R.D.	6 ,,	H.H.	8 28 39·39	22 22 14·14
127	,, 21	W.Z.C. 63	6·9	D.D.	6 ,,	H.H.	5 47 41·29	18 30 56·04
128	,, 28	b^1 Geminorum	5·0	D.D.	6 ,,	H.H.	7 22 44·85	19 38 12·67
129	Mar. 13	τ Sagittarii	3·4	R.D.	6 ,,	H.H.	14 59 37·46	2 26 39·54
130	,, 28	W.Z.C. 558	6·7	D.D.	6 ,,	H.H.	10 41 24·33	21 6 14·18

No.	Star's Apparent R.A.	Star's Apparent Dec.	Moon's R.A.	Moon's Dec.	Moon's Hor. Eq. Parallax	From N. Point: Disappearance	From N. Point: Direction of Motion
	h m s	° ′ ″	h m s	° ′ ″	″	° ′	° ′
87	19 11 10·10	−25 22 56·0	19 7 42·11	−25 38 33·5	3327·06	81 39	87 23
88	21 38 41·05	−19 11 33·5	21 35 34·78	−19 26 45·7	3432·85	98 6	72 29
89	19 32 21·76	−25 2 39·3	19 34 40·71	−25 32 44·2	3327·79	64 36	84 9
90	17 17 38·26	−24 55 53·6	17 16 28·67	−25 18 17·4	3254·24	315 50	98 52
91	15 16 52·66	−17 54 14·3	15 18 35·44	−18 18 11·3	3294·41	304 33	110 6
92	8 6 10·98	+25 43 33·6	8 7 22·39	+25 10 1·1	3533·45	181 27	100 8
93	...	...	...	...	...	...	...
94	15 10 34·82	−17 30 26·8	15 8 57·63	−17 32 32·0	3277·06	172 9	110 52
95	15 16 53·62	−17 54 17·5	15 18 6·96	−18 21 26·2	3273·07	52 48	110 10
96	14 46 46·43	−15 42 20·3	14 48 54·46	−16 4 31·9	3327·89	108 39	112 11
97	14 46 57·88	−15 45 1·4	14 49 9·79	−16 6 2·0	3327·68	115 5	112 10
98	16 21 21·14	−23 17 14·4	16 21 15·32	−23 20 16·1	3270·39	120 12	104 17
99	21 58 36·22	−17 18 17·8	22 0 49·38	−17 35 11·7	3312·26	124 45	68 28
100	7 21 23·44	+27 56 24·0	7 21 59·37	+26 53 10·7	3667·49	46 55	96 2
101	2 44 35·52	+17 59 38·5	2 46 49·71	+17 5 45·9	3422·85	19 32	67 40
102	10 3 32·44	+17 6 14·9	10 6 9·24	+16 41 38·2	3633·96	168 7	112 39
103	12 38 7·92	− 1 4 3·9	12 36 52·81	− 1 40 22·5	3565·57	284 17	117 52
104	12 38 8·18	− 1 4 8·7	12 36 53·21	− 1 40 25·6	3565·57	284 11	117 52
105	22 49 47·34	−11 59 25·0	22 48 3·50	−12 15 8·0	3287·91	234 30	64 36
106	12 6 7·47	+ 2 17 19·3	12 9 12·53	+ 2 0 17·2	3515·96	177 29	117 57
107	12 49 37·50	− 3 10 33·3	12 49 59·57	− 3 24 21·4	3497·35	184 57	117 49
108	23 13 16·27	− 8 6 15·1	23 12 14·35	− 8 39 36·7	3265·95	48 40	62 54
109	...	...	...	...	...	...	...
110	5 0 19·01	+27 36 11·0	4 59 25·94	+26 55 24·9	3471·14	246 23	80 52
111	3 45 3·85	+23 50 46·6	3 46 27·80	+23 3 31·3	3460·58	73 29	73 14
112	3 45 5·16	+23 55 47·2	3 46 41·74	+23 4 29·2	3460·75	55 51	73 16
113	10 28 31·71	+14 29 29·5	10 26 33·52	+13 37 34·6	3616·05	316 42	114 58
114	3 42 13·95	+23 44 14·5	3 41 51·81	+23 6 15·1	3414·97	128 54	73 8
115	3 41 43·35	+24 9 22·6	3 41 56·84	+23 6 36·1	3415·06	16 41	73 9
116	3 43 15·02	+23 54 24·1	3 43 10·33	+23 11 42·0	3416·31	106 56	73 16
117	...	...	...	...	...	...	...
118	3 43 23·17	+24 4 45·4	3 43 24·19	+23 12 39·4	3416·52	71 28	73 17
119	...	...	...	...	...	...	...
120	3 43 23·01	+23 53 45·4	3 43 27·22	+23 12 51·9	3416·58	113 18	73 18
121	...	...	...	...	...	...	...
122	3 42 52·78	+24 18 37·9	3 44 21·73	+23 16 36·7	3417·48	358 12	73 23
123	...	...	...	...	...	...	...
124	3 44 23·52	+24 8 15·9	3 44 55·12	+23 18 53·8	3418·06	77 6	73 27
125	...	...	...	...	...	...	...
126	10 3 35·80	+17 5 56·1	10 2 44·12	+16 5 7·1	3673·49	331 11	113 39
127	1 3 1·23	+ 7 59 28·6	1 5 24·58	+ 7 14 35·9	3261·82	29 1	62 23
128	7 25 4·10	+28 15 50·4	7 23 54·29	+27 17 8·7	3571·54	67 40	98 33
129	19 2 37·89	−27 46 29·8	19 0 42·57	−27 59 15·6	3315·25	255 7	84 6
130	8 6 4·32	+26 3 2·0	8 7 30·12	+25 28 8·4	3547·71	134 59	103 28

No.	Date	Star	Mag.	Phase	Instrument	Obs.	Cape Sid. Time	G.M.T.
	1931						h m s	h m s
131	Mar. 29	90 H^1 Cancri	6·1	D.D.	10-inch	C.O.	12 31 53·56	22 52 29·34
132	May 21	b^2 Geminorum	5·0	D.D.	6 ,,	H.H.	9 47 29·62	16 40 9·19
133	,, 25	308 B Leonis	5·8	D.D.	6 ,,	H.H.	10 24 12·80	17 1 2·71
134	,, 25	B.D. +7° 2440	6·6	D.D.	6 ,,	H.H.	15 29 8·49	22 5 8·44
135	June 1	X Sagittarii	4·3	R.D.	6 ,,	H.H.	13 31 12·14	19 40 0·03
136	,, 27	169 B Libræ	6·0	D.D.	6 ,,	H.H.	21 26 27·51	1 55 39·73
137	,, 27	σ Scorpii	3·1	D.D.	10 ,,	L.D.	12 52 59·66	17 19 40·03
138	July 4	39 Aquarii	6·2	R.D.	6 ,,	H.H.	0 6 7·09	4 7 21·76
139	Oct. 4	183 B Aurigæ	6·3	R.D.	6 ,,	H.H.	2 20 11·50	0 49 15·60
140	Dec. 6	86 Virginis	5·6	R.D.	6 ,,	H.H.	8 11 32·46	2 2 1·57

Notes

1, 10, 14, 15, 24, 30, 40, 55, 61, 62, 65, 66, 67, 83, 98, 101, 110, 112, 113, 116, 117, 118, 119, 120, 124, 125, 132, 136. Conditions poor.
18, 59, 91. Gradual.
25, 86, 140. Time altered 1^m.
62. Time altered 3^m.
75. Strong earthshine.
84. A sudden diminution in brightness 2 sec. before extinction.
100. Near full Moon.
140. Uncertain.

No.	Star's Apparent R.A.	Star's Apparent Dec.	Moon's R.A.	Moon's Dec.	Moon's Hor. Eq. Parallax	From N. Point: Disappearance χ	From N. Point: Direction of Motion P
	h m s	° ′ ″	h m s	° ′ ″	″	° ′	° ′
131	9 9 42·59	+21 34 8·5	9 11 34·11	+21 2 9·3	3598·28	130 34	109 50
132	7 25 31·66	+28 3 45·2	7 26 41·04	+27 7 15·7	3478·01	52 23	99 1
133	11 10 28·13	+ 8 26 13·4	11 9 29·91	+ 8 2 3·9	3564·42	163 47	117 25
134	11 17 55·41	+ 7 0 41·4	11 20 15·02	+ 6 38 28·9	3567·26	145 14	117 45
135	17 43 15·71	−27 48 31·4	17 41 3·30	−28 14 38·3	3458·96	300 41	93 22
136	15 33 46·90	−22 55 6·0	15 36 54·82	−23 9 10·6	3498·29	160 53	106 55
137	16 17 2·13	−25 25 58·2	16 14 13·82	−25 26 8·2	3481·84	178 3	103 4
138	22 8 45·11	−14 31 54·0	22 11 3·57	−14 41 29·4	3282·29	228 48	65 24
139	6 2 1·13	+29 31 17·9	6 0 28·44	+28 43 23·0	3385·24	289 38	89 34
140	13 42 17·37	−12 5 6·7	13 39 59·32	−12 40 30·3	3602·19	284 48	115 44

Notes

19. Star place from *Abbadia Zone Catalogue of* 14263 *Stars.*
43. ,, ,, ,, *First Bergedorf Catalogue*, 1925.
48. ,, ,, ,, *Algiers Zone Catalogue of* 9997 *Stars.*
116. ,, ,, ,, *First Cape Catalogue*, 1925.
118. ,, ,, ,, *Greenwich Catalogue*, 1910, pt. ii.
122. ,, ,, ,, ,, ,, ,, ,,
124. ,, ,, ,, ,, ,, ,, ,,
127. ,, ,, ,, *Cape Catalogue*, 1900, pt. i.
130. ,, ,, ,, *First Cape Catalogue*, 1925.
134. ,, ,, ,, *Cincinnati Catalogue*, No. 19, 1900.

The reductions have been made by the method of Innes (*U.O.C.*, Nos. 55 (1922), 56 (1922), and 63 (1925)), which is so widely used for the reductions of occultations that further explanation is not necessary. The resulting equations of condition are given in Table II in the form

$$\sigma' - \sigma = \cos\delta \sin\chi\Delta\alpha + \cos\chi\Delta\delta$$
$$= \cos(\chi - P)\Delta\lambda - \sin(\chi - P)\Delta\beta.$$

The angles χ and P are given in Table I. Where an occultation has been observed by two observers the mean of the times has been used in the reduction, except in one instance where both times have been reduced.

TABLE II

Resulting Equations of Condition

No.	Date	R.A. and Dec.					Long. and Lat.		
		$\sigma' - \sigma$	=	$\cos\delta \sin\chi\Delta\alpha$	+ $\cos\chi\Delta\delta$	=	$\cos(\chi - P)\Delta\lambda$		$-\sin(\chi - P)\Delta\beta$
	1923								
1	Jan. 20	−0·09	=	+ ·276$\Delta\alpha$	+ ·961$\Delta\delta$	=	+ ·551$\Delta\lambda$	+	·834$\Delta\beta$
2	,, 25	−0·64	=	+ ·976	− ·090	=	+ ·942	−	·335
3	Aug. 8	−0·86	=	− ·737	− ·631	=	− ·821	−	·571
4	,, 20	−0·04	=	+ ·440	− ·887	=	+ ·552	−	·834
5	Sept. 14	−1·22	=	+ ·766	+ ·622	=	+ ·603	+	·798
6	,, 15	0·00	=	+ ·961	− ·101	=	+ ·996	+	·090
7	,, 16	−0·09	=	+ ·701	+ ·679	=	+ ·649	+	·761
8	Oct. 17	−0·17	=	+ ·787	− ·574	=	+ ·714	−	·700
9	,, 18	+0·23	=	+ ·846	− ·494	=	+ ·736	−	·677
10	Dec. 26	−0·09	=	− ·949	− ·195	=	− ·918	−	·397
11	,, 27	−0·38	=	− ·628	+ ·767	=	− ·820	+	·573
12	,, 29	+0·15	=	− ·655	− ·751	=	− ·403	−	·915
	1924								
13	Feb. 1	−2·39	=	− ·709	− ·667	=	− ·669	−	·743
14	,, 7	+1·79	=	+ ·902	+ ·425	=	+ ·993	+	·120
16	,, 14	+1·01	=	+ ·673	+ ·708	=	+ ·786	+	·618
17	,, 26	−0·42	=	− ·731	+ ·655	=	− ·875	+	·484
18	Apr. 10	+0·80	=	+ ·796	+ ·540	=	+ ·858	+	·514
19	,, 10	−0·06	=	+ ·785	− ·556	=	+ ·814	−	·581
20	,, 22	+0·96	=	− ·942	− ·179	=	− ·941	−	·339
21	July 5	+0·65	=	+ ·970	− ·056	=	+ ·983	+	·186
22	,, 14	+0·13	=	+ ·867	− ·397	=	+ ·944	−	·330
23	,, 18	−0·29	=	− ·966	− ·148	=	− ·993	+	·118
24	,, 19	+0·02	=	− ·662	− ·743	=	− ·863	−	·505
25	Aug. 4	+1·47	=	+ ·920	− ·388	=	+ ·998	−	·064
26	,, 6	−2·24	=	+ ·779	− ·622	=	+ ·941	−	·338
27	,, 9	−1·61	=	+ ·144	− ·989	=	+ ·328	−	·945
28	,, 12	−0·63	=	+ ·388	+ ·912	=	+ ·476	+	·879
29	,, 20	+0·44	=	− ·539	− ·840	=	− ·778	−	·628
30	Oct. 3	−1·66	=	+ ·580	+ ·792	=	+ ·503	+	·865
31	Nov. 4	−0·33	=	+ ·284	+ ·956	=	+ ·523	+	·852
32	Dec. 4	+1·66	=	+ ·941	− ·334	=	+ ·774	−	·633
33	,, 4	+0·76	=	+ ·981	− ·184	=	+ ·862	−	·506
	1925								
34	Apr. 13	+1·32	=	− ·914	+ ·265	=	− ·994	+	·111
35	,, 29	+0·06	=	+ ·367	− ·921	=	+ ·469	−	·883
36	May 13	−0·09	=	− ·913	− ·234	=	− ·989	−	·148
37	,, 13	−0·96	=	− ·613	− ·762	=	− ·761	−	·649
38	,, 15	−0·37	=	− ·926	− ·278	=	− ·999	−	·037
39	,, 20	−0·30	=	− ·989	− ·085	=	− ·967	+	·255
40	Aug. 1	−1·37	=	+ ·908	− ·243	=	+ ·983	−	·186
41	,, 3	−0·56	=	+ ·921	− ·210	=	+ ·944	−	·331
42	,, 25	+1·21	=	+ ·463	− ·882	=	+ ·718	−	·696
43	,, 25	+1·49	=	+ ·921	− ·343	=	+ ·999	−	·042

No.	Date	R.A. and Dec. $\sigma' - \sigma$	=	$\cos\delta \sin\chi\Delta\alpha$	+	$\cos\chi\Delta\delta$	Long. and Lat. =	$\cos(\chi - P)\Delta\lambda$	$-\sin(\chi - P)\Delta\beta$
	1925								
44	Aug. 25	−0·67	=	+ ·916$\Delta\alpha$	+	·357$\Delta\delta$	=	+ ·782$\Delta\lambda$	+ ·623$\Delta\beta$
45	,, 25	−1·39	=	+ ·769	+	·619	=	+ ·561	+ ·828
46	,, 29	−0·45	=	+ ·185	+	·980	=	+ ·194	+ ·981
47	Sept. 5	−0·04	=	− ·996	+	·047	=	− ·919	+ ·394
48	Oct. 24	+0·13	=	+ ·744	+	·612	=	+ ·867	+ ·498
49	,, 24	−0·47	=	+ ·707	+	·659	=	+ ·836	+ ·549
50	Nov. 2	+0·72	=	− ·419	−	·898	=	− ·628	− ·779
51	,, 22	+1·56	=	+ ·825	−	·522	=	+ ·676	− ·737
52	Dec. 5	−1·30	=	− ·830	+	·471	=	− ·943	+ ·332
53	,, 24	−1·26	=	+ ·516	+	·854	=	+ ·784	+ ·620
54	,, 28	+1·41	=	+ ·775	−	·562	=	+ ·750	− ·662
	1926								
55	Jan. 22	+1·42	=	+ ·945	+	·217	=	+ ·998	− ·067
56	Feb. 17	+0·37	=	+ ·472	+	·878	=	+ ·748	+ ·664
57	Apr. 17	+1·36	=	+ ·928	−	·005	=	+ ·998	− ·068
58	May 2	−0·20	=	− ·856	+	·379	=	− ·921	+ ·390
	1927								
59	Jan. 11	−0·90	=	− ·171	+	·985	=	+ ·200	+ ·980
60	,, 14	+1·02	=	+ ·932	−	·068	=	+ ·968	− ·252
61	,, 14	−0·32	=	+ ·695	+	·666	=	+ ·847	+ ·531
62	,, 15	+2·78	=	+ ·679	−	·677	=	+ ·665	− ·747
63	Apr. 7	+0·19	=	+ ·708	+	·635	=	+ ·817	+ ·576
65	,, 8	+0·32	=	+ ·480	−	·851	=	+ ·554	− ·833
66	June 5	−1·58	=	+ ·883	−	·389	=	+ ·996	− ·088
67	Oct. 14	+0·07	=	− ·217	−	·973	=	− ·476	− ·879
	1928								
68	Jan. 14	0·00	=	− ·896	+	·435	=	−1·000	+ ·024
69	,, 31	+0·51	=	+ ·911	−	·223	=	+ ·890	− ·457
71	Mar. 17	−0·13	=	− ·899	+	·151	=	− ·964	+ ·264
72	Apr. 9	−0·17	=	+ ·925	+	·181	=	+ ·892	+ ·452
73	,, 9	+1·69	=	− ·590	+	·778	=	− ·821	+ ·571
74	,, 9	+0·75	=	− ·605	+	·765	=	− ·832	+ ·555
75	,, 23	+1·88	=	+ ·907	+	·090	=	+ ·998	− ·056
76	May 6	−0·68	=	− ·904	+	·232	=	−1·000	− ·015
77	,, 6	−0·31	=	− ·904	+	·232	=	−1·000	− ·015
78	,, 7	+1·43	=	+ ·583	−	·770	=	+ ·753	− ·658
79	,, 7	+2·11	=	− ·897	−	·178	=	− ·944	− ·329
81	,, 9	+3·63	=	+ ·902	+	·013	=	+ ·999	+ ·054
82	,, 10	−0·15	=	− ·892	+	·146	=	− ·981	+ ·192
83	,, 12	+0·68	=	− ·900	−	·251	=	−1·000	− ·005
84	June 29	−1·79	=	+ ·641	+	·731	=	+ ·447	+ ·895
85	,, 29	+0·80	=	+ ·615	+	·756	=	+ ·413	+ ·911
86	July 8	+0·50	=	− ·602	−	·791	=	− ·879	− ·478
87	,, 30	−0·65	=	+ ·892	+	·145	=	+ ·995	+ ·100
88	Aug. 29	−1·79	=	+ ·934	−	·141	=	+ ·902	− ·432
89	Sept. 23	−1·38	=	+ ·815	+	·429	=	+ ·942	+ ·335

No.	Date	R.A. and Dec. $\sigma' - \sigma$	=	$\cos\delta \sin\chi \Delta\alpha$	+	$\cos\chi \Delta\delta$	Long. and Lat. =	$\cos(\chi - P)\Delta\lambda$	$-\sin(\chi - P)\Delta\beta$
	1929								
90	Mar. 4	+0·65	=	− ·630	+	·717	=	− ·799	+ ·601
91	,, 29	+1·26	=	− ·782	+	·567	=	− ·968	+ ·250
92	Apr. 16	−1·16	=	− ·023	−	1·000	=	+ ·151	− ·989
94	June 18	−1·03	=	+ ·130	−	·991	=	+ ·480	− ·877
95	,, 18	−0·15	=	+ ·756	+	·605	=	+ ·539	+ ·842
96	Aug. 11	−0·19	=	+ ·911	−	·320	=	+ ·998	+ ·062
97	,, 11	−0·80	=	+ ·870	−	·424	=	+ ·999	− ·051
98	,, 13	−2·85	=	+ ·793	−	·503	=	+ ·962	− ·274
99	Oct. 14	−0·44	=	+ ·783	−	·570	=	+ ·555	− ·832
100	Dec. 18	+4·11	=	+ ·651	+	·683	=	+ ·655	+ ·756
	1930								
101	Mar. 5	+1·55	=	+ ·320	+	·942	=	+ ·667	+ ·745
102	,, 13	+1·33	=	+ ·197	−	·979	=	+ ·567	− ·824
103	,, 15	−1·41	=	− ·969	+	·247	=	− ·972	− ·235
104	,, 15	−1·41	=	− ·969	+	·245	=	− ·972	− ·237
105	Apr. 24	−0·53	=	− ·796	−	·581	=	− ·985	− ·175
106	May 9	+2·25	=	+ ·044	−	·999	=	+ ·507	− ·862
107	,, 9	+0·33	=	− ·086	−	·996	=	+ ·389	− ·921
108	Nov. 1	+0·22	=	+ ·742	+	·660	=	+ ·969	+ ·246
110	,, 8	+2·05	=	− ·817	−	·401	=	− ·968	− ·250
111	Dec. 5	+1·24	=	+ ·882	+	·284	=	+ 1·000	− ·004
112	,, 5	+2·44	=	+ ·761	+	·561	=	+ ·954	+ ·299
	1931								
113	Jan. 7	+1·07	=	− ·667	+	·728	=	− ·929	+ ·370
114	,, 28	+1·01	=	+ ·716	−	·628	=	+ ·563	− ·827
115	,, 28	−0·57	=	+ ·264	+	·958	=	+ ·552	+ ·834
116	,, 28	+1·18	=	+ ·879	−	·291	=	+ ·832	− ·554
118	,, 28	+1·18	=	+ ·871	+	·318	=	+ ·999	+ ·032
120	,, 28	+0·58	=	+ ·844	−	·396	=	+ ·766	− ·643
122	,, 28	+0·28	=	− ·029	+	1·000	=	+ ·256	+ ·967
124	,, 28	−0·06	=	+ ·895	+	·223	=	+ ·998	− ·064
126	Feb. 3	+1·43	=	− ·463	+	·876	=	− ·793	+ ·609
127	,, 21	−0·75	=	+ ·481	+	·874	=	+ ·835	+ ·550
128	,, 28	+0·36	=	+ ·822	+	·380	=	+ ·858	+ ·513
129	Mar. 13	+1·81	=	− ·853	−	·257	=	− ·988	− ·156
130	,, 28	+2·52	=	+ ·639	−	·707	=	+ ·852	− ·523
131	,, 29	+1·56	=	+ ·709	−	·650	=	+ ·935	− ·354
132	May 21	−0·11	=	+ ·705	+	·610	=	+ ·687	+ ·727
133	,, 25	+0·65	=	+ ·276	−	·960	=	+ ·690	− ·724
134	,, 25	+0·98	=	+ ·567	−	·821	=	+ ·887	− ·461
135	June 1	−0·45	=	− ·758	+	·510	=	− ·888	+ ·459
136	,, 27	+0·09	=	+ ·301	−	·945	=	+ ·588	− ·809
137	,, 27	−0·28	=	+ ·031	−	·999	=	+ ·259	− ·966
138	July 4	+0·33	=	− ·728	−	·659	=	− ·958	− ·286
139	Oct. 4	−0·19	=	− ·826	+	·336	=	− ·939	+ ·343
140	Dec. 6	−2·03	=	− ·943	+	·255	=	− ·982	− ·190

The number of occultations observed in any one year is not sufficient to give a strong determination of the corrections to the tabular longitude and latitude. The equations of condition for each year have nevertheless been solved by least squares and the resulting corrections derived. These are summarised in Table III along with the corresponding weights: the adopted correction to the mean longitude has been inserted, so that the total correction is given in the table.

TABLE III

Solutions of Normal Equations

Year	$\Delta\lambda$	Wt.	$\Delta\beta$	Wt.	No. of Obs.
	″		″		
1923	+7·45	6·6	−0·14	5·3	12
24	+7·63	14·1	+0·05	6·5	21
25	+7·08	14·2	−0·87	6·8	21
26	+7·65	3·4	−0·03	0·6	4
27	+6·99	4·9	−0·91	3·9	9
28	+6·34	17·8	−0·57	4·2	22
29	+5·42	5·5	+1·46	4·9	11
30	+5·71	8·5	−0·61	3·2	12
31	+5·39	17·2	−0·35	8·9	28

The probable error of a single observation (unit weight) is found to be ± 0″·79.

The weighted mean value for $\Delta\beta$ is −0″·23. The longitude corrections have been derived from the normal equations for $\Delta\lambda$, assuming a constant value for $\Delta\beta$, with the two assumptions (i) $\Delta\beta = -0''{\cdot}23$, the mean value derived from the present series; (ii) $\Delta\beta = -0''{\cdot}50$, the mean value derived from the rediscussion of Newcomb's occultation memoir.* The values so obtained, with the probable errors, are summarised in Table IV.

TABLE IV

Longitude Corrections

Year	$\Delta\lambda$ ($\Delta\beta = -0''{\cdot}23$)	$\Delta\lambda$ ($\Delta\beta = -0''{\cdot}50$)	p.e.
	″	″	″
1923	+7·46	+7·48	± 0·31
24	7·66	7·68	0·21
25	7·07	7·07	0·21
26	7·65	7·65	0·44
27	6·89	6·93	0·36
28	6·35	6·35	0·19
29	5·07	5·02	0·34
30	5·57	5·60	0·27
31	5·41	5·36	0·19

It will be seen that the longitude corrections derived with the two assumptions as to the value of $\Delta\beta$ are in very close agreement. The general decrease, during the period of observation, of the longitude correction required by Brown's tables is well indicated.

* *Cape Annals*, **13**, pt. iii, p. 54, 1932.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE NATIONAL OBSERVATORY OF ATHENS.

(Communicated by Dr. S. Plakidis)

No.	Star B.D.	Mag.	Date	Local Sidereal Time	No.	Star B.D.	Mag.	Date	Local Sidereal Time
	°		1932	h m s		°		1932	h m s
1	+23 541	2·9	Jan. 18	2 32 18·2	19	+27 1278	9·4	Mar. 16	10 50 7·8
2	+28 932	9·5	Mar. 15	7 40 36·8	20	+27 1279	9·4	,, 16	10 59 23·4
3	+28 935	9·5	,, 15	7 47 44·3	21	+27 1283	9·4	,, 16	11 15 20·3
4	+28 930	8·2	,, 15	8 2 52·3	22	+27 1282	8·3	,, 16	11 17 29·8
5	+28 938	8·8	,, 15	8 19 33·4	23	+27 833	9·3	Apr. 11	10 6 26·5
6	+28 940	8·5	,, 15	8 26 4·5	24	+27 820	9·3	,, 11	10 39 4·9
7	+28 957	8·4	,, 15	9 55 30·2	25	+27 822	9·3	,, 11	10 45 26·6
8	+28 958	8·7	,, 15	10 17 10·4	26	+27 823	9·0	,, 11	10 56 38·8
9	+28 961	9·3	,, 15	10 38 36·6	27	+28 840	9·5	,, 11	11 5 19·7
10	+28 963	9·4	,, 15	10 49 15·3	28	+28 847	9·5	,, 11	11 38 50·7
11	+28 964	9·2	,, 15	10 52 3·4	29	+27 831	9·2	,, 11	11 41 13·5
12	+28 966	6·9	,, 15	11 10 21·9	30	+28 848	9·4	,, 11	11 42 20·5
13	+27 1255	9·4	,, 16	8 58 40·9	31	+28 850	9·0	,, 11	11 57 18·0
14	+27 1258	8·5	,, 16	9 35 26·1	32	+28 853	8·9	,, 11	12 7 49·4
15	+27 1263	9·1	,, 16	9 40 14·7	33	+28 854	9·0	,, 11	12 9 57·8
16	+27 1267	9·2	,, 16	9 52 43·0	34	+13 2239	8·6	,, 16	12 38 39·5
17	+27 1266	9·2	,, 16	10 4 17·4	35	− 7 3503	8·0	,, 19	9 44 11·8
18	+27 1271	9·1	,, 16	10 34 0·5					

Instrument used :

Doridis refractor (Gautier, 400 mm.), except in the case of No. 1 (η Tauri) which was observed with the finder (80 mm.) through clouds.

Magnifying power :

155 times.

Phase observed :

Only disappearances at the dark limb.

Remarks :

The time of No. 35 is somewhat uncertain.

Observer :

G. Adamopoulos.

National Observatory of Athens :
1932 *May* 5.

OCCULTATIONS OF STARS BY THE MOON, OBSERVED AT THE ROYAL OBSERVATORY OF TURIN (PINO TORINESE) DURING THE YEAR 1931.

M. A. Ferrero.

(Communicated by the Secretaries)

No.	Star	Mag.	Phase	Date	G.C.T.	χ	$\sigma' - \sigma$	$\chi - \rho$
				d	h m s	° ′	″	° ′
1	χ Tauri	5·3	D	Feb. 25	18 48 17·1	32 29	−1·0	− 44 31
2	B.D. +25° 707	9·0	D	,, 25	18 49 26·2	31 8	−1·4	− 45 52
3	107 B Aurigæ	6·5	D	,, 27	1 28 55·5	62 17	−1·5	− 23 14
4	ξ Arietis	4·8	D	Mar. 23	18 48 39·3	69 28	−0·4	− 0 53
5	406 B Tauri	5·6	D	Apr. 22	20 0 51·2	94 30	−0·7	+ 7 11
6	B.D. +28° 918	8·1	D	,, 22	20 19 42·1	62 7	−2·1	− 25 17
7	B.D. +27° 891	8·4	D	,, 22	20 23 18·4	118 45	−0·2	+ 31 20
8	B.D. +27° 895	9·4	D	,, 22	20 52 22·5	76 41	−0·6	− 10 53
9	27 B Virginis	6·5	D	June 22	21 8 18·7	68 43	+1·1	− 49 41
10	α Virginis	1·2	D	July 21	19 32 54·4	82 51	−1·1	− 33 56
11	α Virginis	1·2	R	,, 21	20 27 24·7	339 19	+1·5	−137 23 *
12	φ Capricorni	5·3	D	Dec. 13	16 42 50·8	93 26	−1·6	+ 24 4
13	φ Capricorni	5·3	R	,, 13	17 43 57·0	202 22	+4·5	+133 10 *
14	B.D. −20° 6178	6·8	D	,, 13	18 40 15·5	123 21	+0·9	+ 54 20
15	B.D. −5° 6011	7·1	D	,, 16	17 6 25·7	30 48	−2·8	− 31 0
16	W.Z.C. 55	7·8	D	,, 18	21 16 49·9	352 46	−0·6	− 69 46
17	ϵ Piscium	4·4	D	,, 18	21 19 21·5	40 56	−1·9	− 21 36
18	χ Tauri	5·3	D	,, 23	1 46 59·7	149 55	+0·8	+ 71 34

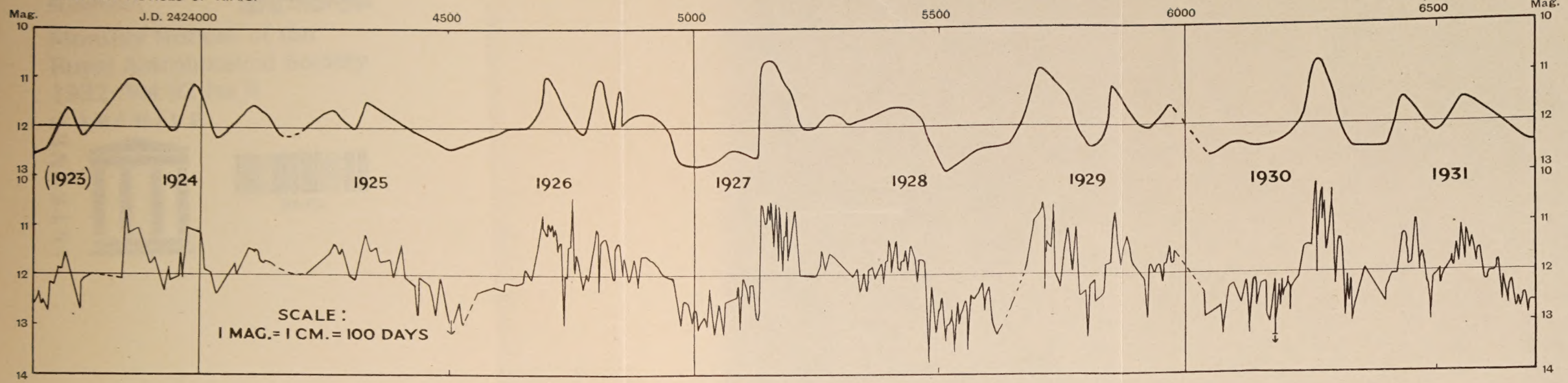
* Bright limb.

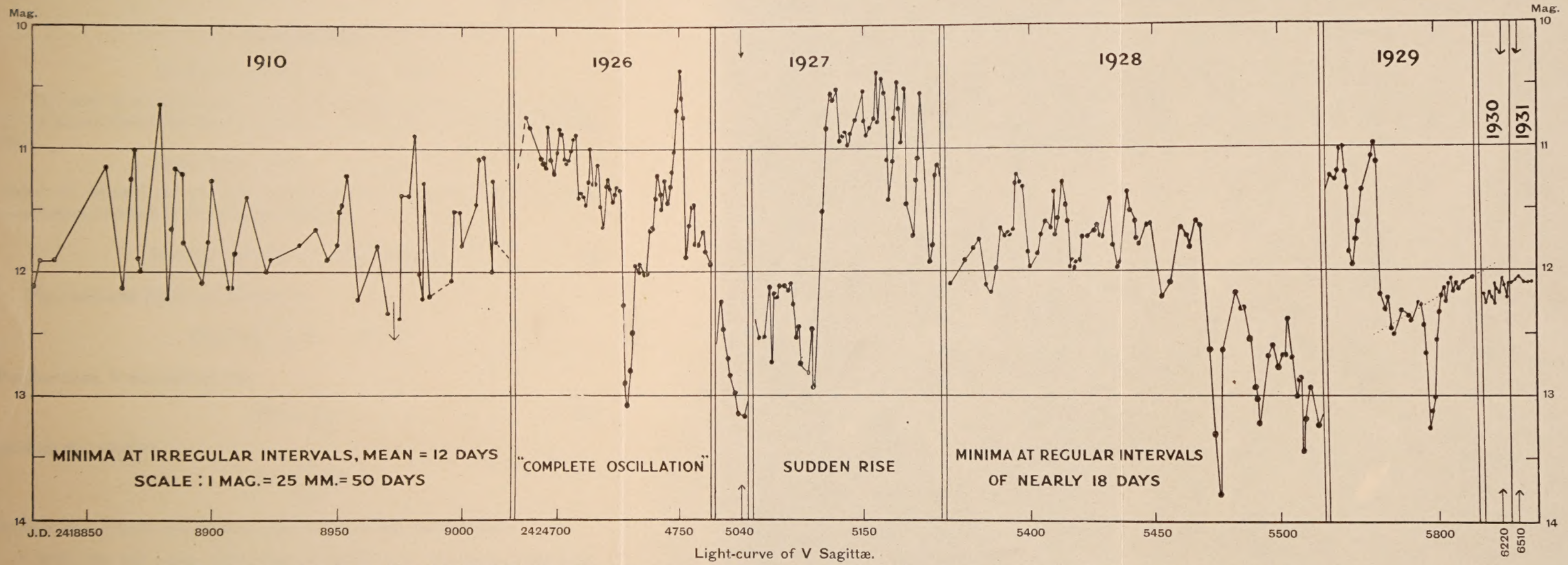
The observations were made with the 30-cm. Merz refractor. The times were determined with a chronograph from the normal clock of the observatory (Riefler). A correction of $-0^s{\cdot}1$ or $-0^s{\cdot}2$ was made for personal error. The observations were reduced by the observer by two different methods. The usual correction of $+0^h{\cdot}0030$ was made to the times as given above for an error of $+6''{\cdot}0$ in the mean longitude of the Moon.

Turin :
1932 *April.*

Errata.

P. 617, footnote, *for* " Mr. G. Lillie " *read* " Mr. G. Cillié."
P. 632, on lines 3, 6, 12, *for* T *read* S′.
P. 634, footnote ‡, line 2, *for* " control " *read* " central."
,, ,, line 2, *for* " temperature ρ' " *read* " density ρ'."





Light-curve of V Sagittæ.

The upper part of the diagram is based on 1100 observations made at Zaragoza (Spain). The time-scale is very closed up. Drastic smoothing was employed in the top curve in order to reveal the general tendency of long-term variation.

The lower part of the diagram shows portions of the above on a larger and more open scale, also part of an earlier series. Note in 1929 the "abortive maximum" and the case of a narrow minimum not affecting the slow rise.

[*See p.* 715.